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THE PATH MATRIX AND SWITCHING FUNCTIONS *

BY

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ABSTRACT

Part I introduces a new matrix, the path matrix, in the theory of linear graph. The matrix is defined and its properties are given in a number of lemmas and theorems. They include (1) a relation between the path matrix and the incidence matrix of a connected graph (Theorem 1) and (2) the rank of the path matrix, which is shown to be $e - v + 2 - S$ (Theorem 3), where e is the number of edges, v the number of vertices and S the number of independent circuits contained in the "path-isolated subgraphs" of a connected graph. It is clear that there is a one-to-one correspondence between the union of all paths between two vertices and a two-terminal switching function. An explicit formula is obtained which relates a switching function to the topology of the graph. The formula shows that a switching function is expressible in terms of the path matrix in much the same manner as the determinant of the admittance matrix is expressed in terms of the vertex matrix.

Part II demonstrates a synthesis procedure which obtains a graph from a given path matrix without first converting it into a circuit matrix by means of the "free element" as did Okada, Seshu and Gould. The procedure is based on a number of relations between paths and cut sets, which are derived in the paper. As there is a one-one correspondence between a path matrix and a switching function of n variables, the procedure is directly applicable without modification to the synthesis of a switching function of n variables using n contacts.

PART I

I. INTRODUCTION

Part I introduces the path matrix in the theory of linear graphs. The matrix is defined and its properties are presented in a number of lemmas and theorems, which have been found useful in the analysis and synthesis of switching functions. This is not unexpected, for there is obviously a one-to-one correspondence between a two-terminal switch-

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ing function of n variables and the union of all paths between two vertices in a graph of n or more edges. Other applications suggest themselves and some will certainly be found in areas of network theory where paths between two specified vertices play the dominant role (1, 2).[2]

Mayeda (3) studied the properties of paths and found the maximum number of independent paths in a connected graph. The minimum number was not given. Okada (4, 5), Seshu (6) and Gould (7) all had applied linear graph theory to the study of two-terminal switching circuits, and hence indirectly to paths in a graph. However, they all add to the graph a "free element," which connects the input and the output vertices, so that the union of all paths becomes the union of all circuits that contain this free element. Instead of dealing with paths directly, they deal with circuits, whose properties are well known.

It is the opinion of some that the properties of paths can be derived from the properties of circuits. This may be true, but it is not as straightforward as one might at first suspect. (See Conclusion to Part I.)

Part I of this paper derives the properties of the path matrix and paths without recourse to the free element. Some of the important properties include: (1) a relation between the path matrix and the incidence matrix (Theorem 1) and (2) the rank of the path matrix, which is shown to be $e - v + 2 - S$ (Theorem 3), where e is the number of edges, v the number of vertices and S the number of independent circuits contained in the "path-isolated" subgraphs of the connected graph. These properties are used to obtain an expression of the hindrance function (complement of a switching function) in terms of the path matrix in much the same manner as the determinant of an admittance matrix is expressed in terms of the vertex matrix.

II. DEFINITIONS AND THEOREMS

Some knowledge of network topology is assumed. Seshu (8) gives an excellent summary of the properties of linear graphs. Only non-oriented graphs will be considered in this paper.

Definition 1: A *path* is a connected, ordered sequence of edges whose terminal vertices (the first and the last relative to the ordering) are of degree 1 and whose internal vertices are of degree 2.

Definition 2: A *path matrix*, denoted by P, of a connected graph of v vertices and e edges is a matrix whose e columns correspond to the edges of the graph and whose rows correspond to all paths between the first vertex V_1 and the last vertex V_v. The elements of P, p_{ij}, are determined as follows:

$$p_{ij} = 1 \qquad \text{if edge } j \text{ is in path } i;$$

$$p_{ij} = 0 \qquad \text{if edge } j \text{ is not in path } i.$$

[2] The boldface numbers in parentheses refer to the references appended to this paper.

As an example, the path matrix of Fig. 1 is given by:

$$P = \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}. \tag{1}$$

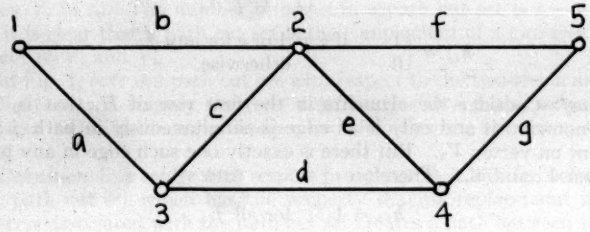

FIG. 1.

Let us write the incidence matrix A of the graph of Fig. 1, with the columns of A arranged in the same edge order as in P.

$$A = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}. \tag{2}$$

Let us post-multiply A by the transpose of P (mod. 2).

$$AP^t = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}. \tag{3}$$

It is seen that the product matrix consists of two rows of 1's, the remaining elements being all zero, a result also observed by Okada (4). These two rows correspond to the two terminal vertices V_1 and V_5. We therefore have the following theorem:

Theorem 1: The product of the incidence matrix A and the transpose of the path matrix P of a connected graph, when the columns in both, which correspond to the edges of the graph, are arranged in the same order, is a matrix having exactly two rows of 1's, the remaining elements being all zero. The two vertices corresponding to the two rows of 1's are the terminal vertices of the paths in P. That is,

$$AP^t = H \equiv [h_{ij}] = \begin{bmatrix} 1 & 1 \cdots 1 \\ 0 & 0 \cdots 0 \\ 0 & 0 \cdots 0 \\ 0 & 0 \cdots 0 \\ 1 & 1 \cdots 1 \end{bmatrix} \quad \text{(mod. 2)}$$

$$h_{ij} = \begin{cases} 1 & \text{if } i = 1 \text{ or } v \text{ for any } j; \\ 0 & \text{otherwise.} \end{cases}$$

Proof: Consider the elements in the first row of H, that is, h_{1j}. h_{1j} is non-zero if and only if an edge is simultaneously in path j and incident on vertex V_1. But there is exactly one such edge in any path between V_1 and V_v. Therefore

$$h_{1j} = 1 \qquad \text{for all } j.$$

Similarly

$$h_{vj} = 1 \qquad \text{for all } j.$$

For $i \neq 1$ or v, we note that along any path, the vertices that are not the terminal vertices are all of degree 2 or 0. Therefore $h_{ij} = 0$ in mod. 2, for $i \neq 1$ or v for any j.

Theorem 1 plus other properties of the path matrix permits us to prove that the rank of the path matrix is $e - v + 2 - S$ as stated in Section I. To this end, we need several lemmas, which follow.

Lemma 1: The rank of a path matrix cannot exceed $e - v + 2$, where e is the number of edges and v the number of vertices in a connected graph.

Proof: Let there be n paths. Then H is of order $v \times n$. The nullity of A is $e - v + 1$. The nullity of H is $n - 1$. The nullity of P^t is $n -$ (rank of P). Applying Sylvester's law of nullity on the product of matrices, we have from $AP^t = H$

$$\text{nullity of } A + \text{nullity of } P^t \geq \text{nullity of } H.$$

Substitution yields

$$\text{Rank of } P \leq e - v + 2.$$

Lemma 1 has the significance that the set of all paths between two

specified vertices can be generated by linear combinations (mod. 2) of not more than $e - v + 2$ linearly independent paths, a result first obtained by Mayeda in a different manner (3).

As an example, with reference to the path matrix of Fig. 1, the path *adg* is the mod. 2 sum of the last three paths. In fact all paths can be generated by the following four $(e - v + 2 = 4)$ independent ones: *bf*, *adg*, *bcdg*, and *beg*. Clearly there is more than one set of independent paths in Fig. 1.

Definition 3: A *path cut set* is a set of edges which have the property that their removal from the graph G leaves G with a two-tree (9), which consists of two disjoint subtrees t_1 and t_2, with vertex V_1 in t_1 and vertex V_ν in t_2. The number of edges in a path cut set is $e - v + 2$, and it is clear that a path cut set is the complement of a two-tree with respect to V_1 and V_ν.

In Fig. 1, *bcde* is a path cut set with respect to the two-tree *a* and *fg*.

Next we will classify the $(e - v + 2)$ edges in a path cut set into two sets. One is the set of *bridges*, and the other the set of *non-bridge edges*.

Definition 4: A *bridge* with respect to a two-tree is an edge belonging to a path cut set which has the property that its replacement in the two-tree associated with the path cut set creates a path between vertex V_1 and vertex V_ν. All other edges in the path cut set are *non-bridge edges*, and they are the chords associated with the subtrees t_1 and t_2. For example *e* is the non-bridge edge in the path cut set *bcde*, whose removal cuts the graph into two circuit-less subgraphs, *a* and *fg*. It is clear that a path cut set of a connected graph must contain at least one bridge.

Lemma 2: Between vertex V_1 and vertex V_ν, each and every bridge in a path cut set defines a unique path, which consists of exclusively this bridge and some or all edges of t_1 and t_2. The set of paths defined by the bridges form an independent set.

This lemma is obvious, for if there exists another such path, t_1 or t_2 will have to contain a circuit. This is impossible.

The last property allows us to write the path matrix P_b corresponding to the paths defined by the set of bridges in the following manner:

$$P_b = [uP_{12}]$$

where u is a unit matrix of order k (k = number of bridges in a path cut set).

Definition 5: A *path-isolated subgraph* with respect to V_1 and V_ν in a connected graph G is a separable subgraph of G whose removal from G removes neither V_1 nor V_ν. It is clear that none of the edges of a path-isolated subgraph is contained in any path between V_1 and V_ν. Figure

2*a* shows a graph which contains a path-isolated subgraph. Figure 2*b* shows a separable graph which does not contain a path-isolated subgraph.

Lemma 3: In a connected graph which does not contain any path-isolated subgraphs there is a unique path associated with each non-bridge edge, which includes that non-bridge edge, exactly one bridge, possibly some other non-bridge edges and some or all branches of t_1 and t_2. The set of $e - v + 2 - k$ such paths form an independent set.

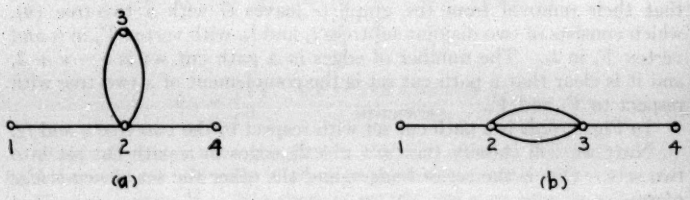

FIG. 2.

Proof: Replace one (any one) bridge b to the two-tree t_1 and t_2. Call the resultant graph G'. The replacement of a non-bridge edge of G' may or may not create a path between V_1 and V_2. If it does, this path is obviously unique and it will consist of this non-bridge edge, the bridge b and some or all edges of t_1 and t_2. Group all non-bridge edges which have this property into a set S. If the replacement does not create a path, add to G' some edges in S such that their replacement to G' will create a path which contains the non-bridge edge not in S. This path is obviously unique. Now since the graph contains no path-isolated subgraphs, there are no non-bridge edges which are not contained in some path, and the number of unique paths is $e - v + 2 - k$.

As an illustration of Lemma 3, consider Fig. 3. The path matrix corresponding to the set of non-bridge edges can be constructed according to the proof of the lemma as follows:

Bridges		*Non-Bridge Edges*				*Two-Tree*				
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
1	0	1	0	0	0	0	1	1	0	0
1	0	0	1	0	0	0	1	0	0	0
1	0	1	0	1	0	1	0	0	0	1
1	0	0	1	0	1	1	0	1	0	0

The bridge in G' is a. The set S consists of edges c and d. In general, it is clear that the path matrix, P_{nb}, corresponding to the $e - v + 2 - k$ non-bridge edges can be partitioned as follows:

$$P_{nb} = [P_{11} \quad P_{12} \quad P_{13}]$$

where P_{12} is a triangular matrix of order $e - v + 2 - k$ whose diagonal elements are all 1.

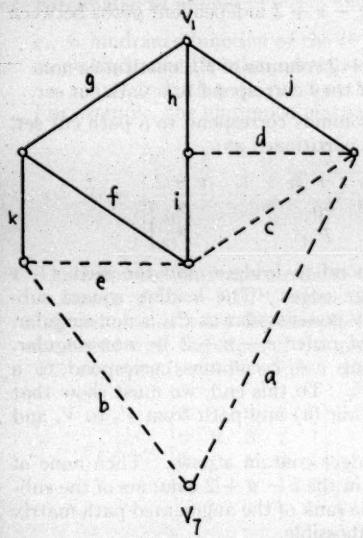

FIG. 3.

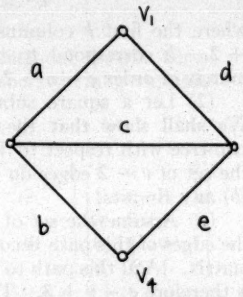

FIG. 4.

A direct consequence of Lemmas 2 and 3 is:

Lemma 4: The rank of the path matrix of a connected graph which contains no path-isolated subgraphs is at least $e - v + 2$.

A direct consequence of Lemmas 1 and 4 is:

Theorem 2: The rank of the path matrix of a connected graph which contains no path-isolated graphs is $e - v + 2$.

Removing the restriction of "path-isolated subgraphs," we have:

Theorem 3: The rank of the path matrix of a connected graph is $e - v + 2 - S$, where S is the number of independent circuits in the path-isolated subgraphs of the graph.

Proof: The chords associated with the trees in the path-isolated subgraphs are not contained in any path. Therefore the expression $e - v + 2$ must be reduced by the number of such chords. But there

are as many such chords as there are independent circuits in the path-isolated subgraphs. Hence the theorem.

In the sequel, only graphs which contain no path-isolated subgraphs will be considered.

Definition 6: A *fundamental path matrix*, denoted by P_f, is a path matrix whose rows correspond to $e - v + 2$ independent paths between V_1 and V_v.

Theorem 4: Any set of $e - v + 2$ columns of P_f constitute a non-singular submatrix if and only if they correspond to a path cut set.

Proof: (1) Let the $e - v + 2$ columns correspond to a path cut set. From Lemmas 2 and 3, P_f can be partitioned as:

$$P_f = \begin{array}{c} \begin{matrix} k & e-v+2 & v-2 \end{matrix} \\ \begin{bmatrix} u & 0 & P_{13} \\ P_{21} & P_{22} & P_{23} \end{bmatrix} \end{array}$$

where the first k columns correspond to bridges, and the next $e - v + 2 - k$ correspond to non-bridge edges. The leading square submatrix of order $e - v + 2$ is clearly non-singular as P_{22} is non-singular.

(2) Let a square submatrix of order $e - v + 2$ be non-singular. We shall show that the remaining $v - 2$ columns correspond to a two-tree with respect to V_1 and V_v. To this end, we must show that the set of $v - 2$ edges do not contain (*a*) any path from V_1 to V_v and (*b*) any circuits.

(*a*) Assume the set of $v - 2$ edges contain a path. Then none of the edges of this path is contained in the $e - v + 2$ columns of the submatrix. Add this path to P_f. The rank of the augmented path matrix is therefore $e - v + 3$. This is impossible.

(*b*) A path by definition does not contain any circuit. No subset, the $v - 2$ edges included, of the set of edges in a path contains any circuit.

A set of $v - 2$ edges that contains no paths between V_1 and V_v nor circuits must be a two-tree with respect to V_1 and V_v. Its complement, the set of $e - v + 2$ edges, must therefore be a path cut set.

III. SWITCHING FUNCTION IN TERMS OF A PATH MATRIX

The properties of the path matrix are now used to derive an explicit formula between a two-terminal switching function and the topology of the circuit. Suppose in Fig. 4 we treat the edge weights $(a, b, \cdots, e)$ as the five variables of a switching function between V_1 and V_4. The switching function is

$$f = ab + de + ace + dcb.$$

The complement of f, the hindrance function (10), g, is:

$$g = f' = (a' + b')(d' + e')(a' + c' + e')(d' + c' + b')$$

which is seen to be the product of the sum of edge weights of all paths. The form of g suggests that we can express it as the determinant of some matrix.

Definition 7: A *hindrance matrix*, G, is a symmetric matrix of the order of the number of all paths between V_1 and V_2 whose elements g_{ij}, are determined as follows:

g_{ii} = hindrance function of the i^{th} path;
g_{ij} = hindrance function corresponding to the edges that are common to both path i and path $j = g_{ji}$.

For example, the hindrance matrix of Fig. 4 is:

$$G = \begin{bmatrix} a'+b' & 0 & a' & b' \\ 0 & d'+e' & e' & d' \\ a' & e' & a'+c'+e' & c' \\ b' & d' & c' & d'+c'+b' \end{bmatrix}.$$

The following lemma is obvious.

Lemma 5: $G = PEP^t$, where E is a diagonal matrix whose diagonal elements are the complements of the switching variables associated with the edges of the graph.

For example, it can be verified that in Fig. 4:

$$G = PEP^t = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} a' & & & & \\ & b' & & O & \\ & & c' & & \\ & O & & d' & \\ & & & & e' \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Theorem 5: The permanent of G (in Boolean and not in mod. 2 algebra) is the hindrance function g.

Proof: The off diagonal elements of G are included (in Boolean sense) in the elements of the diagonal. When the permanent of G is expanded, the terms that are multiplied by the off diagonal elements are all included in the one term which is the product of the diagonal elements, which is the hindrance function g.

Definition 8: A *basic cut set* is the set of bridges in a path cut set.

Definition 9: A *path-cut-set product* is the product of the *complement* of the edge weights of the edges in a path cut set.

Definition 10: A *basic-cut-set product* is the product of the *complement* of the edge weights of the edges in a basic cut set. It is clear that since a path cut set contains a basic cut set, the path-cut-set product implies the basic-cut-set product in Boolean sense. The following theorem is well known. We shall prove it using the properties of the path matrix.

Theorem 6: The hindrance function of a two-terminal swtiching circuit is the sum of the basic-cut-set products.

Proof: In the expression g = permanent (PEP^t), we expand the right side by the Binet-Cauchy Theorem (11). Recalling Theorem 4, we have

$$g = \text{sum of all path-cut-set products.}$$

Since each path-cut-set product implies a basic-cut-set product, we have

$$g = \text{sum of all basic-cut-set products}$$

as asserted.

For example, in Fig. 3, we have the sum of all basic-cut-set products $= a'd' + b'e' + a'c'e' + d'c'b'$. From the expansion of permanent (PEP^t), we find the path-cut-set products to be: $a'd'e' + b'e'c' + a'c'e' + d'c'b'$. The first two terms imply, respectively, the basic-cut-set products $a'd'$ and $b'e'$.

If we expand the permanent of the matrix G, we find after simplification by Boolean algebra,

$$g = a'd' + b'e' + a'c'e' + d'c'b'$$

which is the sum of all basic-cut-set products as asserted in the theorem.

IV. CONCLUSION TO PART I

It has been suggested that the rank of the path matrix can be deduced directly from the circuit matrix which is obtained by appending a column of 1's to the path matrix. Assume the graph has no path-isolated subgraphs. The augmented matrix obviously has a rank which cannot exceed $e - v + 2$. In order to show that the rank is at least $e - v + 2$, one must show that there exist $e - v + 2$ independent circuits among the rows of the augmented matrix. However, suppose we add to the graph a free element, which connects the input and output vertices. Ashenhurst (12) has shown that if the augmented graph is not separable, any circuit not containing the free element can be generated by mod. 2 combinations of the set of paths between the two vertices. Therefore, the set of circuits containing this free element contains a basis for the set of all circuits. The number of independent paths is therefore $e - v + 2$. His lemma can be modified to include the case where the graph contains path-isolated subgraphs.

On the other hand, this paper proves *by construction* that the number of independent paths is at least $e - v + 2 - S$ for *any* connected graphs. In other words we have given a systematic method of constructing a set of independent paths, which the other method of proof (using the free element) does not give.

It should be noted that the path-isolated subgraphs do occur in switching circuits of more than two terminals. In such cases, a subgraph may be "path-isolated" with respect to vertices 1 and 2 but not with respect to, say, 1 and 3.

There are other properties of paths in a connected graph. They relate the path matrix and the cut set matrix and are important in the synthesis of a two-terminal switching function of n variables using n contacts. The synthesis procedure as well as the other properties will be given in Part II of this paper.

PART II

V. INTRODUCTION

In Part I, we introduced a new matrix in the theory of linear graphs. It is the path matrix, whose properties we derived and applied to the analysis of a two-terminal switching circuit.

In Part II, we wish to demonstrate a synthesis procedure which obtains a graph from a given path matrix without first converting it into a circuit matrix by means of the "free element" as did Okada (4), Seshu (6) and Gould (7). The procedure is based on a number of relations between the paths in a graph and the "basic" and "non-basic" cut sets. These relations are derived in Part II, as are other important properties of cut sets and cut set matrices.

Noting the one-one correspondence between a path matrix and a two-terminal switching function of n variables, we remark without further comments that the synthesis procedure is directly applicable to the realization of a two-terminal switching function of n variables using n contacts.

VI. DEFINITIONS AND THEOREMS

Definition 11: A *basic cut set* is a cut set whose edges have the property that their removal from the graph separates the graph into two disjoint subgraphs with V_1 (the first vertex) in one and V_v (the last vertex) in the other. A cut set which does not have this property shall be called a *non-basic cut set*. For example, in Fig. 5, with respect

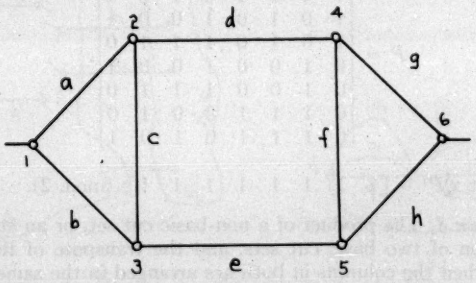

FIG. 5.

to vertices 1 and 6, (*acfh*) is a basic cut set and (*acfg*) a non-basic cut set.

The following relations between cut sets and paths are obvious from the definitions and are important in devising a synthesis procedure to realize a path matrix (a switching function).

Lemma 6: A basic cut set with respect to vertices V_1 and V_* has an odd number of edges in common with a path between these vertices.

Lemma 7: A non-basic cut set with respect to vertices V_1 and V_* has an even number of edges in common with a path between these vertices.

Lemma 8: An edge-disjoint union of two basic cut sets has an even number of edges in common with a path between V_1 and V_*.

The relations between a path and various other disjoint union of cut sets can all be deduced from the above three lemmas and shall not be listed. The two theorems below follow from the lemmas.

Theorem 7: The product of a basic cut set, or an edge-disjoint union of a basic cut set and a non-basic cut set, and the transpose of the path matrix, when the columns in both, which correspond to the edges of the graph, are arranged in the same order, yields a row of ones (mod. 2).

For example, in Fig. 5, the basic cut set (*acfh*) in matrix form is:

$$x = \begin{matrix}[\,a & b & c & d & e & f & g & h\end{matrix}\\ [1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1]. \tag{4}$$

The path matrix of Fig. 5 is:

$$P = \begin{matrix} a & b & c & d & e & f & g & h \end{matrix}\\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}. \tag{5}$$

The product $x_*^1 P^t = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]$, (mod. 2).

Theorem 8: The product of a non-basic cut set, or an edge-disjoint union of two basic cut sets, and the transpose of the path matrix, when the columns in both are arranged in the same order, yields a row of zeros (mod. 2).

For example, the non-basic cut set $(acgf)$ in Fig. 5 in matrix form is:

$$y = \begin{matrix} a & b & c & d & e & f & g & h \\ [1 & 0 & 1 & 0 & 0 & 1 & 1 & 0]. \end{matrix} \tag{6}$$

The product $y P^t = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$, (mod. 2).

Definition 12: A *fundamental path-cut-set matrix*, denoted by A_{fp}, with respect to a two-tree, which contains two subtrees, t_1 and t_2, with V_1 in t_1 and V_v in t_2, is a matrix whose columns correspond to the edges of the graph and whose $v - 2$ rows (v = number of vertices of the graph) are sets of edges each of which includes (1) exactly one tree branch, (2) bridges (see Part I) whose paths contain this tree branch, and (3) chords of t_1 or t_2 whose circuits contain this tree branch. It is clear that the rank of A_{fp} is $v - 2$ as it contains a unit matrix of order $v - 2$ and there are $v - 2$ rows.

As an example, the fundamental path-cut-set matrix with respect to the two-tree (ab) and (gh) of Fig. 5 is:

$$A_{fp} = \begin{matrix} c & d & e & f & \quad a & b & g & h \\ \end{matrix} \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{7}$$

268
59

Theorem 9: The set of edges defined by the rows of a fundamental path-cut-set matrix with respect to V_1 and V_v all have an even number of edges in common with any path between V_1 and V_v, and are in fact non-basic cut sets, or disjoint union of two basic cut sets. That is, $A_{fp} P^t = 0$, (mod. 2).

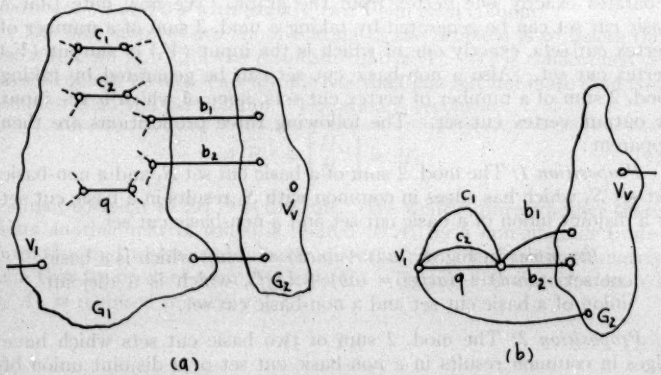

FIG. 6.

Proof: Consider one row of the fundamental path-cut-set matrix. Let the tree branch in question be q. Let the set (possibly empty) of bridges whose paths contain q be $(b_1, b_2, \cdots)$ and the set (possibly empty) of chords whose circuits contain q be $(c_1, c_2, \cdots)$. The graph is shown in Fig. 6*a*. G_1 contains t_1 and its chords, and G_2 contains t_2 and its chords. The bridges whose paths do not contain q are unlabelled. The dotted lines represent edges which are not q or $(c_1, c_2, \cdots)$. Since these edges do not play any role in showing that the set $(q, b_1, b_2, \cdots, c_1, c_2, \cdots)$ has an even number of edges in common with any paths, we shall shrink their lengths to zero. The reduced graph is shown in Fig. 6*b*.

Case 1: The set of unlabelled bridges is not empty. Then removal of $(q, b_1, b_2, \cdots, c_1, c_2, \cdots)$ leaves the graph with two disjoint parts with V_1 and V_* both contained in one part, and no proper subset of the set $(q, b_1, b_2, \cdots, c_1, c_2, \cdots)$ has this property. Therefore the set $(q, b_1, b_2, \cdots, c_1, c_2, \cdots)$ must be a non-basic cut set.

Case 2: The set of unlabelled bridges is empty. Then removal of the set $(q, b_1, b_2, \cdots, c_1, c_2, \cdots)$ leaves the graph with three disjoint parts with V_1 and V_* contained in two separate parts. The set $(q, b_1, b_2, \cdots, c_1, c_2, \cdots)$ is therefore a disjoint union of two basic cut sets. In both cases, the number of edges in common with any path from V_1 to V_* is even by Lemmas 7 and 8.

We shall have occasions to take mod. 2 sums of basic cut sets and non-basic cut sets. We recall that a vertex cut set is a cut set which separates exactly one vertex from the graph. We now note that a basic cut set can be generated by taking a mod. 2 sum of a number of vertex cut sets, exactly one of which is the input (V_1) or output (V_*) vertex cut set. Also a non-basic cut set can be generated by taking mod. 2 sum of a number of vertex cut sets, none of which is the input or output vertex cut set. The following three propositions are then apparent:

Proposition 1: The mod. 2 sum of a basic cut set S_1 and a non-basic cut set S_2 which has edges in common with S_1 results in a basic cut set or a disjoint union of a basic cut set and a non-basic cut set.

Example: In Fig. 5, $(ab) + (acd) = (bcd)$, which is a basic cut set. $(bcd) + (acfg) = (ab) + (dgf)$, which is a disjoint union of a basic cut set and a non-basic cut set.

Proposition 2: The mod. 2 sum of two basic cut sets which have edges in common results in a non-basic cut set or a disjoint union of two basic cut sets.

Example: In Fig. 5, $(ab) + (bcd) = (acd)$, a non-basic cut set, and $(acfh) + (bcfg) = (ab) + (gh)$, a disjoint union of two basic cut sets.

Proposition 3: The mod. 2 sum of two non-basic cut sets which have edges in common results in a non-basic cut set or a disjoint union of two basic cut sets.

Example: In Fig. 5, $(acd) + (dfg) = (acfg)$, a non-basic cut set, and $(acfg) + (bcfh) = (ab) + (gh)$ a disjoint union of two basic cut sets.

Theorems 1, 2, 3 and the propositions enable us to devise a synthesis procedure to realize a graph from a given path matrix. Briefly, we first construct a fundamental path-cut-set matrix from the given path matrix. Next we add to it a row which is a basic cut set. Linear combinations of the rows of the resultant matrix lead to a vertex matrix, from which a graph can be constructed by inspection.

VII. REALIZATION OF A GRAPH FROM ITS PATH MATRIX

From a given path matrix, we shall abstract a fundamental path matrix (see Part I), P_f. The problem then is to solve the following matrix equation, which is given in Part I:

$$AP_f^t = \begin{bmatrix} H_{11} \\ H_{21} \\ H_{31} \end{bmatrix} \quad \text{(mod. 2)} \tag{8}$$

where $H_{11} = H_{31} = [11 \cdots 11]$, and H_{21} is a matrix of zeros, and where A is the vertex matrix, which is constrained to be a matrix whose elements are 1's and 0's with exactly two 1's in each column.

The first step toward simplification of the problem is to delete the last row from A. Call the resultant matrix A_1. A_1 is constrained to have elements 1's and 0's with not less than one but not more than two non-zero elements per column. Equation 8 becomes:

$$A_1 P_f^t = \begin{bmatrix} H_{11} \\ H_{21} \end{bmatrix} \equiv H_1. \tag{9}$$

A direct solution to Eq. 9 is difficult. We shall inquire whether there exists another matrix which is related to A_1 by a non-singular transformation and which is obtainable directly from the given path matrix such that its product with P^t yields H_1. That is, the matrix, denoted by A_c, is required to satisfy:

$$A_c = TA_1 \tag{10}$$

and

$$A_c P_f^t = H_1 \tag{11}$$

where T is a non-singular matrix. From Eqs. 9, 6 and 11, we have:

$$TH_1 = H_1. \tag{12}$$

A nontrivial T must therefore be a triangular matrix of order $v - 1$ having 1's along the main diagonal, 0's below it, and 1's and 0's above it. Such a transformation, when applied to A_1, amounts to elementary row operations on A_1. The matrix A_c is in fact a cut set matrix (6).

The problem now is reduced to one of finding A_c from P_f. To this end, we shall partition A_c and P_f as follows:

$$A_c = \begin{bmatrix} A_{c11} & A_{c12} \\ A_{c21} & A_{c22} \end{bmatrix} \tag{13}$$

where

A_{c11} is (1) by $(e - v + 2)$, e = number of edges of the graph,
A_{c12} is (1) by $(v - 2)$,
A_{c21} is $(v - 2)$ by $(e - v + 2)$, and
A_{c22} is $(v - 2)$ by $(v - 2)$.

$$P_f{}^t = \begin{bmatrix} P_{11}{}^t \\ P_{12}{}^t \end{bmatrix} \tag{14}$$

and P_{11} is a nonsingular matrix of order $e - v + 2$ (see Part I). From Eq. 9, we get:

$$[A_{c11} \quad A_{c12}]P_f{}^t = [11 \cdots 1] \tag{15}$$

$$[A_{c21} \quad A_{c22}]P_f{}^t = 0. \tag{16}$$

From Theorems 7 and 9, we find immediately:

$$A_{c11} = [11 \cdots 1](P_{11}{}^t)^{-1} \tag{17}$$

$$A_{c12} = [00 \cdots 0] \tag{18}$$

$$A_{c21} = P_{12}{}^t(P_{11}{}^t)^{-1} \tag{19}$$

$$A_{c22} = U \tag{20}$$

where U is of order $v - 2$. Stated in words, the matrix $[A_{c11} \quad A_{c21}]$ is a basic cut set, and the matrix $[A_{c21} \quad A_{c22}]$ is the fundamental path-cut-set matrix. The cut set matrix A_c is given by:

$$A_c = \begin{bmatrix} H_{11}(P_{11}{}^t)^{-1} & 0 \\ P_{12}{}^t(P_{11}{}^t)^{-1} & U \end{bmatrix} \tag{21}$$

which satisfies $A_e P_f{}^t = H_1$. Moreover, its rank is clearly $v - 1$ and is related to the matrix A_1 by a non-singular transformation. In general, A_e as given by Eq. 21 is not a vertex matrix and we must find it by row operations on A_e. Propositions 1, 2 and 3 ensure that these operations leave the product of A_e and $P_f{}^t$ invariant.

As an example, consider finding a graph from the following switching function of eight variables using eight contacts:

$$F = adg + adfh + aceh + acefg + beh + befg + bcdg + bcdfh. \quad (22)$$

The corresponding path matrix is given by Eq. 5. The following fundamental path matrix can be abstracted:

$$P_f = \begin{array}{c} \begin{matrix} d & e & c & f & \quad a & b & g & h \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}. \quad (23)$$

The rank of the matrix is 4 so that the number of vertices is $v = 6$. From Eq. 21, we get:

$$P_{12}{}^t = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad (24)$$

$$(P_{11}{}^t)^{-1} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (25)$$

The cut set matrix is found to be

$$A_e = \begin{array}{c} \begin{matrix} d & e & c & f & \quad a & b & g & h \end{matrix} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \quad (26)$$

The non-singular transformation which takes A_e into A_1 is:

$$T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (27)$$

which amounts to changing the first row of A_e by adding the first three rows. The matrix A_1 is given by

$$A_1 = \begin{array}{c} \begin{matrix} d & e & c & f & \quad a & b & g & h \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}. \tag{28}$$

Adding to A_1 the "deleted" row, we get the vertex matrix A, whose corresponding graph is Fig. 5.

Note that the first row of A_e as given by Eq. 26 is a basic cut set. The remaining rows are all non-basic cut sets.

VIII. CONCLUSION TO PART II

We have given an algorithm which obtains a graph from a given path matrix without the artifice of a "free element." The algorithm makes use of the properties of paths and cut sets, and it depends on finding a nonsingular transformation which takes A_e into a realizable A_1. The question of the existence of such a transformation naturally arises. This question is of course intimately connected with the realizability of a path matrix. In other words, given a matrix of elements 1's and 0's, what distinguished features does it have in order that it is the path matrix of a realizable graph? Equivalently, given a switching function of n variables what are the necessary and sufficient conditions for it to be realizable by a graph of n contacts? This is an outstanding unsolved problem in the theory of linear graphs.

If we append a column of 1's to the path matrix (6), the problem can be converted into one of finding the necessary and sufficient conditions on the realizability of a circuit matrix. Some conditions have been found (13) but the complete solution is not yet in sight.

REFERENCES

(1) A. SHIMBEL, "Structure in Communication Nets," Proc. Symp. on Information Networks, Polytechnic Institute of Brooklyn, April, 1954, pp. 199–203.
(2) J. N. WARFIELD, "Switching Circuits as Topological Models in Discrete Probability Theory," (letter) *Trans. IRE*, Vol. EC-5, pp. 251–252 (1958).
(3) W. MAYEDA, "Application of Mathematical Logic to Network Theory," Interim Report No. 9, U. S. Army Contract No. DA-11-022-Ord-1983, University of Illinois, January, 1958.
(4) S. OKADA, "Topology Applied to Switching Circuits," Proc. Symp. on Information Networks, Polytechnic Institute of Brooklyn, April, 1954.
(5) S. OKADA, "Algebraic and Topological Foundation of Network Synthesis," Proc. Symp. on Modern Network Synthesis, Polytechnic Institute of Brooklyn, 1955.
(6) S. SESHU, "On Electric Circuits and Switching Circuits," *Trans. IRE*, Vol. CT-3, pp. 172–178 (1956).

(7) R. L. GOULD, "Application of Graph Theory of the Synthesis of Contact Networks," Proc. Int. Symp. on Switching Circuits, Harvard University, April, 1957.
(8) S. SESHU, "Theory of Linear Graphs with Applications in Electrical Engineering," Syracuse University, 1958.
(9) W. S. PERCIVAL, "Solution of Passive Electrical Networks by Means of Mathematical Trees," *J. Inst. Elec. Engrs.* (London), Vol. 100, Pt. III, pp. 143–150 (1953).
(10) C. E. SHANNON, "The Synthesis of Two-Terminal Switching Circuits," *Bell System Tech. J.*, Vol. 28, pp. 59–98 (1949).
(11) A. C. AITKEN, "Determinants and Matrices," New York, Interscience Publishers, 1939, p. 88.
(12) R. L. ASHENHURST, "A Uniqueness Theorem for Abstract Two-Terminal Switching Networks," Pt. 6, Bell Laboratories Progress Report No. BL-10, Harvard Computation Laboratory, 1955.
(13) R. B. ASH AND W. H. KIM, "On the Realizability of a Circuit Matrix," *Trans. IRE*, Vol. CT-6, pp. 219–233 (1959).

268
59

FINITE DEFLECTIONS OF THIN CAMBERED ELASTIC PLATES

BY

R. L. BISPLINGHOFF [1]

ABSTRACT

A theory is presented for computing the finite deflections of thin cambered elastic plates subjected to simultaneous bending and twisting moments. The relations between deflections and applied moments are nonlinear and are characterized by instabilities which arise as a result of chordwise deformation. Experiments are described which corroborate the theoretical trends.

INTRODUCTION

Ashwell (1)[2] has demonstrated a characteristic type of instability in the large deflections of cambered elastic plates. This instability is associated with finite bending and it represents the familiar snapping action which one experiences in bending a steel measuring tape. Reissner (2) points out a similar type of instability which arises during the finite bending and twisting of flat plates. The two instabilities are coupled since they arise from the same mechanism of chordwise deformation. The present note shows the nature of this coupling action.

THEORY

We consider a solid rectangular cambered plate of length $2a$ and width $2b$, where $a \gg b$, loaded by pure twisting and bending moments designated by M_t and M, respectively. Figure 1 illustrates the axis system and other dimensional notation. The constant plate thickness is represented by h and the camber by $x_3(x_2)$. The latter is taken as an even function of x_2.

Following Reissner (2) the form of the deflection shape is taken as

$$u_3(x_1, x_2) = \theta(x_1 x_2) - \tfrac{1}{2}kx_1^2 + W(x_2) \qquad (1)$$

where θ is the rate of twist, k is the curvature in the x_1 direction and $W(x_2)$ is a function to be determined which describes the chordwise deformation. Inclusion of the chordwise deformation is required in order to account for the types of instability which are of interest.

Since all chordwise sections behave in an identical manner it will be sufficient to treat the bending of an elemental strip cut from the plate by two planes perpendicular to the x_1-axis and a unit distance apart (see Fig. 1). We assume that due to the large bending and twisting

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

deformations of the long cambered strip there are mid-plane stress resultants N_{11} in the x_1 direction. The stress resultants N_{22} in the chordwise or x_2 direction are assumed zero. For a unit element of the chordwise strip we have the equilibrium equations

$$\frac{dQ_2}{dx_2} - N_{11}k = 0, \qquad \frac{dM_{22}}{dx_2} - Q_2 = 0 \tag{2}$$

where Q_2 and M_{22} are shear stress resultants and stress couples, respectively.

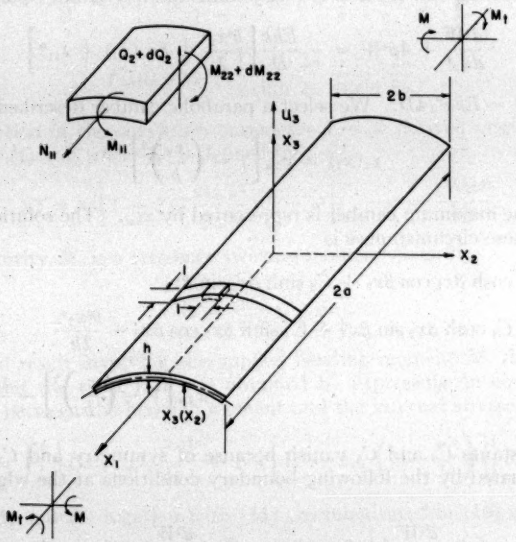

FIG. 1. Cambered rectangular plate.

A relation between M_{22}, W and k may be derived from plate theory, as follows

$$M_{22} = -D\left(\frac{d^2W}{dx_2^2} - \nu k\right) \tag{3}$$

where $D = \dfrac{Eh^3}{12(1-\nu^2)}$ is the flexural rigidity of the plate and ν is Poisson's ratio. Combining (2) and (3) yields

$$D\frac{d^4W}{dx_2^4} + N_{11}k = 0. \tag{4}$$

The stress resultant N_{11} can be expressed in terms of the mid-plane strain $\bar{\epsilon}_{11}$ in the x_1 direction by (3)

$$N_{11} = Eh\bar{\epsilon}_{11} \tag{5}$$

and

$$\bar{\epsilon}_{11} = \frac{\theta^2 x_2^2}{2} + kW + kx_3(x_2) + \bar{\epsilon}_{11}^* \tag{6}$$

where $\bar{\epsilon}_{11}^*$ is the mid-plane strain at $x_2 = \pm b$.

Combining (4), (5) and (6) we obtain the differential equation

$$\frac{d^4W}{dx_2^4} + 4\beta^4 W = -\frac{Ehk}{D}\left[\frac{\theta^2 x_2^2}{2} + kx_3(x_2) + \bar{\epsilon}_{11}^*\right] \tag{7}$$

where $\beta^4 = Ehk^2/4D$. We select a parabolic camber described by

$$x_3(x_2) = x_{3m}\left[1 - \left(\frac{x_2}{b}\right)^2\right] \tag{8}$$

where the maximum camber is represented by x_{3m}. The solution of (7) under these circumstances is

$$W = C_1 \cosh \beta x_2 \cos \beta x_2 + C_2 \sinh \beta x_2 \sin \beta x_2 + C_3 \cosh \beta x_2 \sin \beta x_2 + C_4 \sinh \beta x_2 \cos \beta x_2 - \frac{\theta^2 x_2^2}{2k} - x_{3m}\left[1 - \left(\frac{x_2}{b}\right)^2\right] - \frac{\bar{\epsilon}_{11}^*}{k}. \tag{9}$$

The constants C_3 and C_4 vanish because of symmetry and C_1 and C_2 are evaluated by the following boundary conditions at the edge of the plate:

$$(a)\ \frac{d^3W}{dx_2^3}(\pm b) = 0, \qquad (b)\ \frac{d^2W}{dx_2^2}(\pm b) = \nu k. \tag{10}$$

Combining (9) and (10) gives

$$C_1 = \frac{1}{\beta^2}\left(\frac{\theta^2}{k} + \nu k - \frac{2x_{3m}}{b^2}\right)\frac{\sinh \mu \cos \mu - \cosh \mu \sin \mu}{\sinh 2\mu + \sin 2\mu}$$
$$C_2 = \frac{1}{\beta^2}\left(\frac{\theta^2}{k} + \nu k - \frac{2x_{3m}}{b^2}\right)\frac{\sinh \mu \cos \mu + \cosh \mu \sin \mu}{\sinh 2\mu + \sin 2\mu} \tag{11}$$

where $\mu = \beta b$ is a curvature parameter.

A result connecting the applied twisting moment M_t with the curvature k and the twist rate θ can be obtained by expressing an equilibrium

relation between the twisting moment and the internal stresses in the following way (3):

$$M_t = \theta \int_{-b}^{b} N_{11} x_2^2 dx_2 + 4b(1 - \nu)D\theta. \tag{12}$$

When we substitute (5) into (12) taking into account (6), (9) and (11) we obtain

$$\frac{M_t}{\bar{M}_t} = \lambda^2 \left[1 + \frac{\nu}{1 - \nu} F_1(\mu) + \frac{\lambda_e^4}{1 - \nu} \frac{F_1(\mu)}{\mu^4} \right] \tag{13}$$

where

$$F_1(\mu) = 1 - \frac{1}{\mu} \frac{\cosh 2\mu - \cos 2\mu}{\sinh 2\mu + \sin 2\mu} \tag{14}$$

is a function of the curvature parameter $\mu = \beta b$ derived originally by Reissner (1) and where λ_e^4 is defined by

$$\lambda_e^4 = \lambda^4 \left[1 - 2\sqrt{3} \frac{x_{3m}}{h} \frac{\mu^2}{\lambda^4} \sqrt{1 - \nu^2} \right], \qquad \lambda^4 = \frac{Ehb^4\theta^2}{4D}.$$

The quantity $\bar{M}_t$ is a reference twisting moment given by

$$\bar{M}_t = \frac{4Dh}{\sqrt{3}b} \sqrt{\frac{1 - \nu}{1 + \nu}}. \tag{15}$$

A second result involving the applied bending moment M, the curvature k and the twist rate θ is obtained by expressing an equilibrium relation between the bending moment and the internal stresses (3):

$$M = \int_{-b}^{b} \left\{ Dk - D\nu \frac{d^2W}{dx_2^2} + N_{11} \left[W + x_{3m} \left(1 - \frac{x_2^2}{b^2} \right) \right] \right\} dx_2. \tag{16}$$

When (5) and (9) together with (11) are substituted in (16) we obtain

$$\frac{M}{\bar{M}} = \frac{\mu^2}{1 - \nu^2} \left[1 - \nu^2 \left(1 - F_1(\mu) - \tfrac{1}{4}\mu \frac{dF_1(\mu)}{d\mu} \right) + \frac{\nu\lambda_e^4}{2\mu^3} \frac{dF_1(\mu)}{d\mu} - \frac{\lambda_e^8}{\mu_8} \left(F_1(\mu) - \tfrac{1}{4}\mu \frac{dF_1(\mu)}{d\mu} \right) - 2\sqrt{3} \frac{x_{3m}}{h} \sqrt{1 - \nu^2} \left(\nu + \frac{\lambda_e^4}{\mu^4} \right) \frac{F_1(\mu)}{\mu^2} \right] \tag{17}$$

where $\bar{M}$ is a reference bending moment defined as

$$\bar{M} = \frac{2Dh\sqrt{1 - \nu^2}}{\sqrt{3}b}. \tag{18}$$

Equations 13 and 17 reduce to Reissner's (2) results when x_{3m} is put equal to zero and Eq. 17 to Ashwell's (1) result with λ_c put equal to zero.

The nature of the solutions represented by (13) and (17) are illustrated by Figs. 2 and 3. These figures apply to solid cambered plates

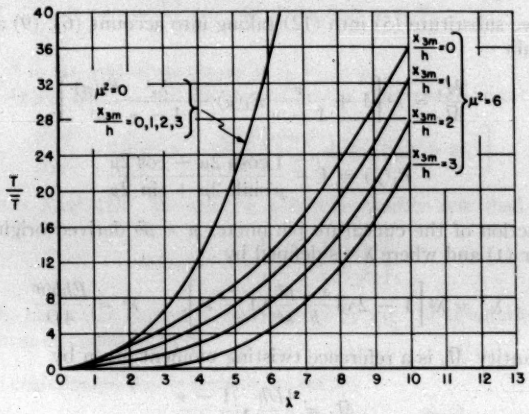

FIG. 2. Influence of camber and bending curvature on the twisting behavior.

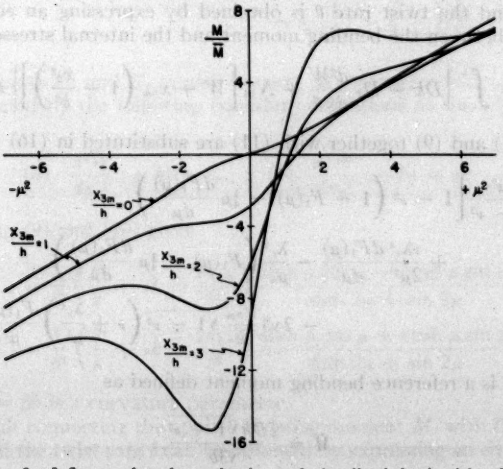

FIG. 3. Influence of camber and twist on the bending behavior ($\lambda^2 = 3$).

of constant thickness having maximum camber to thickness ratios, x_{3_m}/h, of 0, 1, 2 and 3. As already pointed out by Ashwell (1), the influence of camber is to produce a nonlinearity in the curves of bending moment versus curvature. The effects on twisting behavior of bending curvature are illustrated by Fig. 2. Here we observe that camber has no influence on the twisting behavior as long as the beam curvature remains zero. Adding positive or downward beam curvature, as illus-

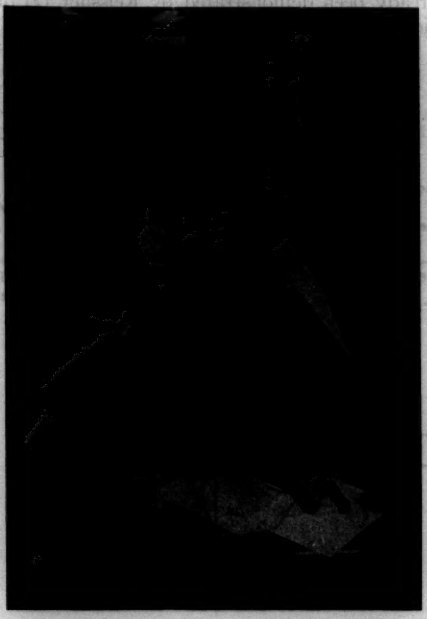

FIG. 4. Loaded specimen.

trated by Fig. 2, produces a reduction of torsional stiffness, with the plate having the greatest camber showing the largest reduction of stiffness. The latter effect can be explained by observing that during bending the cambered plate is subjected to compressive stresses near the edges and tensile stresses in the vicinity of the mid-chord. During the process of twisting through large angles these stresses produce twisting moment components which tend to increase further the twist angle and hence reduce the apparent torsional stiffness. The addition of negative or upward beam curvature would by analogous reasoning increase the torsional stiffness.

Figure 3 shows the influence of camber and twist on the bending behavior of cambered plates. One effect of twist is to move the bending moment curvature curves for cambered plates away from the origin. This is due simply to the fact that finite twist induces tensile stresses at the edges of the plate and compressive stresses in the vicinity of the mid-chord. Their effect is to produce a finite bending curvature in the absence of applied bending moment. The other effect of twist is that which has already been demonstrated by Reissner (2) of producing a snapping action in the bending behavior.

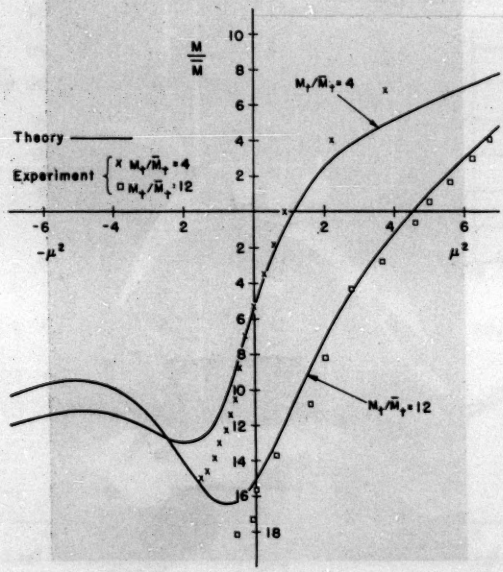

FIG. 5. Theory and experiment for combined bending and twisting of a cambered plate.

THEORY AND EXPERIMENT

In order to compare theoretical results with experiment, tests were conducted on a cambered plate having maximum camber-to-thickness ratio, x_{3m}/h of 3. The loading device employed for these experiments is illustrated by the photograph of Fig. 4.

Figure 5 shows a comparison between theory and experiment for combined bending and twisting. Bending tests were conducted for the two applied twisting moments of $M_t/\bar{M}_t = 4$ and 12. The trend of the theory is supported by the experimental data. The quantitative

agreement is fair. Imperfections in plate camber and inaccuracies associated with the relatively simple loading device are probably responsible for some portion of the deviation between theory and experiment. No attempt was made to compare experiment with theory after buckling took place since the buckling was violent and the amplitudes after buckling exceeded the capabilities of the measuring devices.

REFERENCES

(1) D. G. ASHWELL, "A Characteristic Type of Instability in the Large Deflections of Elastic Plates," *Proc. Roy. Soc., A*, Vol. 214, pp. 98–118 (1952).

(2) ERIC REISSNER, "Finite Twisting and Bending of Thin Rectangular Elastic Plates," *J. Appl. Mech.*, Vol. 24, pp. 391–396 (1957).

(3) R. L. BISPLINGHOFF, "The Finite Twisting and Bending of Heated Elastic Lifting Surfaces," Mitteilung Nr. 4 aus dem Institut für Flugzeugstatik und Leichtbau an der Eidgenössischen Technischen Hochschule in Zürich.

QUENCHING EFFECTS AND THE AFTERGLOW OF CHLOROPHYLL *

BY

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ABSTRACT

The weak light-emission of *Chlorella pyrenoidosa* about 4 seconds after illumination by photosynthesizing blue light shows an apparent partial quenching when radiation at λ7000 Å is simultaneously applied. This result accords with the present view that this afterglow arises from the thermal release of electrons trapped in a crystalline structure of chlorophyll. The "quenching" radiation would provide optical release. Phosphorescence excited by longer wavelengths than blue is not quenched, suggesting it to arise by an essentially different mechanism. Chlorophylls and derivatives dissolved in a film of cellulose butyrate show a rather characteristic weak afterglow, often of several minutes half-life, of shorter wave-length than the fluorescence, and which is not certainly a light-induced oxidative chemiluminescence.

INTRODUCTION

A relatively short-lived red afterglow was observed in experiments of 1935 when solutions of chlorophyll were irradiated by short-wave light (1).[4] More recently, Strehler and co-worker (2) have observed a phosphorescence of longer life-time (seconds or minutes) from green plants (chlorella suspensions) and isolated chloroplasts from spinach. Subsequently, extensive additional studies (3, 4, 5) have been carried out to determine the absorptive and emissive properties of chlorophyll present in green plants and chloroplasts.

In the present investigations, the phosphorescence of chlorella, extracted chlorophyll in organic solvents, and chlorophyll dissolved in a lacquer film have been studied, using a photomultiplier tube as the detector. Measurements were limited to the long-lived afterglow referred to above.

PROCEDURE

Chlorella was grown in a 2.5-liter pyrex culture flask in a modified Knopf solution at room temperature. This solution was shaken vigorously while irradiated by the light of four 20-watt blue fluorescent tubes immediately beneath the flask. Atmospheric air and CO_2 (partial

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pressure about 5 per cent) were circulated continuously from a carboy through the flask. The chlorella suspension itself was circulated by a peristaltic pump through a fused quartz cell (5-cm. diameter, height 1 cm.) which had been place against the window of a red sensitive photomultiplier tube (E.M.I. 6255B). The effective circulation time from flask to cell was about 4 seconds.

For excitation of the phosphorescence, three phases of culture illumination were employed involving blue light, red light, and a combination of the two. Red light was obtained from a 100-watt ruby glass lamp and red fluorescent lamps with a ruby glass filter, while blue light was produced by covering blue fluorescent lamps with two sheets of medium blue-green glass and one of cobalt blue, the sheet thickness being in each case about 3 mm. By spectroscopic observation, it was determined that the blue and red sources did not overlap in frequency, that is, the red radiation lay on the long-wave side of Hg 5790 Å, while the blue radiation was of shorter wavelength than Hg 5460 Å.

In the case of observations upon the phosphorescence of living chlorella, the suspension was irradiated in, and circulated from, the culture flask, while shaking. The afterglow of the film of lacquer, which had been colored by addition of chlorophyll, was observed by painting a spot 5 cm. in diameter upon an aluminum disc, irradiating the spot, and turning it before the photomultiplier in a light-tight arrangement. The conductivity of this lacquer film was about 5×10^{-15} mho/cm. On illuminating a film of thickness 0.0036 cm. by a 100 watt bulb at a distance of 8 cm., the conductivity was increased, if at all, by less than 1×10^{-15} mho/cm. Conducting glass served as one electrode, a layer of mercury being the other. A potential of 22.5 volts was applied.

RESULTS

The afterglow phenomena described in the preceding paragraphs can be demonstrated under either blue or red excitation. The two bands of excitation frequencies were also applied simultaneously with interesting results. The after glow of active chlorella or chloroplast preparations was first brought to saturation intensity under blue illumination. While this blue excitation was being continued, red light was also applied at its saturation level. The net phosphorescent intensity was found not to be the sum of the two effects but essentially that of red alone. This result was particularly striking, since at times, the red-excited phosphorescent saturation intensity was as little as 60 per cent of the blue-excited saturation intensity.

Attempts have been made to isolate the quenching wavelengths with the aid of filters. The quenching effect of the ruby glass bulb was not reduced by the interposition of a cell of water. Insertion of cobalt blue glass or red-purple ultraviolet transmitting glass (Corning 5874) gave only a moderate reduction in the quenching effect as did a

solution of mixed chlorophylls. However, a solution of iodine in aqueous sodium iodide appeared to absorb the quenching radiation completely. The chlorophyll and iodine solutions give high transmission for infrared radiation beyond wavelengths of 7250 Å and 7550 Å, respectively. Transmission curves for these solutions and other filters employed are given in Fig. 1.

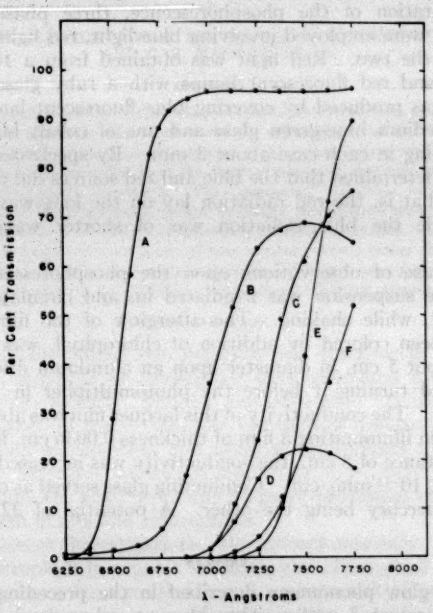

FIG. 1. Transmission of filters.

A, Corning dense red, No. 2–64, 4.5 mm. in thickness.
B, Cobalt Glass, 2 mm.
C, Mixed chlorophylls, 24 mm.
D, Corning N. R. red-purple ultra, No. 5874, 5 mm
E, Corning Theatre Blue, No. 5030, 7.5 mm.
F, NaI–I_2 solution, 24 mm.

A, *B*, *C*, and *D* transmit the quenching light freely to moderately. *D* was used with a 250-watt red heat lamp as the purest convenient source of quenching radiation.

These experiments suggest that radiation as short in wavelength as about 7000 Å is effective in quenching. This quenching radiation is thus not in the far infrared, but lies within about 400 Å of the fluorescent emission itself which is peaked at approximately 6600 Å. The fluorescence of a chlorophyll solution irradiated by ultraviolet was faintly visible through the cobalt and red-purple filters but not through the iodine solution. Thus, the quenching band and the fluorescence band actually appear to overlap somewhat.

The wavelength of the delayed phosphorescent emission was itself investigated by placing filters between the photomultiplier and the quartz cell through which the phosphorescing chlorella was circulated. A ruby glass filter alone permitted appreciable transmission of the radiation associated with the long-lived afterglow, but when employed in combination with a filter of aqueous iodine solution, any observable effect was eliminated. The long-lived afterglow must then have a spectral distribution much the same as the fluorescence as was demonstrated by Arnold and Davidson (3) in the case of the short-lived afterglow.

As previously stated, the quenching effect was initially observed when the phosphorescence, induced by blue excitation, was quenched by simultaneous red illumination. After the quenching band had been located by the previously described filter studies, an additional set of measurements was carried out wherein the red light of a ruby bulb alone was employed as primary excitant of the phosphorescence. This red-excited phosphorescence was then observed to be enhanced rather than quenched when the radiation of a 250-watt heat lamp equipped with a red-purple ultra Corning filter was applied simultaneously.

Since the quenching radiation decreases the intensity of blue excited phosphorescence but increases the intensity of the red excited phosphorescence, it becomes probable that more than one set of traps or metastable states are involved. To test this point, the lifetime of the phosphorescence was measured under various conditions of excitation. The data are summarized in Table I.

TABLE I.—*Lifetime of the Long Period Phosphorescence Under Various Conditions of Excitation.*

Excitant	Lifetime, sec.
Blue fluorescent lamps	4.5
Red fluorescent lamp	5.0
Ruby bulb	8.0
Heat lamp with red-purple ultra Corning filter	11.0

DISCUSSION

Recent measurements of photoionization (7) and paramagnetic resonance (8, 9) have been interpreted as evidence of a crystalline lattice of chlorophyll molecules. These views are incorporated in the hypotheses and summary of the photosynthetic process by Brugger and Franck (10). The quenching of the blue-excited phosphorescence in the present investigations could well be explained as similar to the optical emptying of traps in much the same manner as F-centers are eliminated in crystalline solids. The absence of quenching of red-excited phosphorescence suggests not only that a different set of traps are involved, but also that such metastable levels are not filled by way of the conduction band. The afterglow of the cellulose butyrate lacquers may indicate further mechanisms for a delayed light emission

of chlorophyll, which might conceivably operate *in vivo* without involving the photosynthetic process. By analogy with other organic phosphors this butyrate "glass" has presumably little crystallinity, and as previously indicated, shows little or no photoconductivity.

The possible presence of two or more types of traps or metastable states responsible for long-lived afterglow may conceivably be connected with the greater absorption of carotene in the blue region and its greater transparency than chlorophyll at longer wavelengths. The role of carotene in photosynthesis is being actively investigated at present (11).

Finally, it should be remarked that a number of transient effects of a few seconds duration have been observed. For example, when the quenching radiation is applied, a slight initial increase in afterglow is observed. When the quenching light is extinguished, the phosphorescent intensity returns to a value momentarily exceeding its equilibrium rate.

It is hoped that all of the observations reported in this paper will be correlated ultimately with rate of photosynthesis and other important variables.

NOTE IN PROOF:

Prof. James Franck has kindly pointed out that a true quenching effect need not be assumed, but only a change in shape of the decay-curve in its entirety. This leaves the indications of two mechanisms unaltered, as well as emphasizing the reactive effects of photosynthesis, rather than a quasi-independent source of afterglow.

Franck also suggests an oxidative chemiluminescence set up by irradiation as the explanation of the afterglow of the lacquers. Tests on a variety of them have indicated that red-fluorescing films containing chlorophyll derivatives show stronger and longer afterglow than anything else tried, that the emission is shorter in wavelength than the fluorescence, and that any available oxidizable substrate is much less in amount than in a published case of chemiluminescence (12). A definite conclusion regarding these lacquers is being sought.

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195

REFERENCES

(1) H. KAUTSKY, A. HIRSCH AND W. FLESCH, *Ber. Deut. Chem. Gen.*, Vol. 68, 152 (1935). References to earlier studies of the luminescence of chlorophyll are contained in this paper.
(2) B. L. STREHLER AND W. ARNOLD, *J. Gen. Physiol.*, Vol. 34, p. 809 (1951); B. L. STREHLER, *J. Gen. Physiol.*, Vol. 34, p. 239 (1951).
(3) W. ARNOLD AND J. B. DAVIDSON, *J. Gen. Physiol.*, Vol. 37, p. 677 (1954).
(4) W. E. ARTHUR AND B. L. STREHLER, *Arch. Biochem. Biophys.*, Vol. 70, p. 507 (1957).
(5) B. R. STREHLER AND VICTORIA LYNCH, *Arch. Biochem. Biophys.*, Vol. 70, p. 527 (1957).
(6) E. RABINOWITCH, *J. Phys. Chem.*, Vol. 61, p. 870 (1957).
(7) W. ARNOLD AND H. K. SHERWOOD, *Proc. Natl. Acad. Sci. U. S.*, Vol. 43, p. 105 (1957).
(8) B. COMMONER, J. J. HEISE AND J. TOWNSEND, *Proc. Natl. Acad. Sci. U. S.*, Vol. 42, p. 710 (1956).
(9) P. B. SOGO, N. G. PON AND M. CALVIN, *Proc. Natl. Acad. Sci. U. S.*, Vol. 43, p. 387 (1957).
(10) J. E. BRUGGER AND J. FRANCK, *Arch. Biochem. Biophys.*, Vol. 75, p. 465 (1958).
(11) J. R. PLATT, *Science*, Vol. 129, p. 372 (1959).
(12) A. G. FERRARI, B. L. STREHLER AND W. E. ARTHUR, "Research in Photosynthesis," edited by Hans Gaffron, Interscience Publishers, 1957.

THE DERIVATION OF POLE-ZERO PATTERNS BY DERIVATIVE ADJUSTMENT

BY

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ABSTRACT

The coefficients and pole-zero locations of a transfer function $Z(s)$ containing n poles and m zeros may be determined by imposing a total of $(n + m - 1)$ conditions on the amplitude $A(\omega)$ and the phase $\Phi(\omega)$ of $Z(i\omega)$. In particular, these conditions may be imposed by equating to zero at the origin the first r even derivatives of $A(\omega)$ and the first $(n + m - 1 - r)$ even derivatives of $\Phi'(\omega)$.

Two special cases have been treated in the literature, both applying to transfer functions consisting only of poles ($m = 0$). When $r = n - 1$, the sequence of Butterworth patterns is obtained; $r = 0$ corresponds to the maximally linear phase patterns investigated by Thomson and Storch. The general case is studied in this paper, and is reduced to a set of simultaneous equations in the coefficients of $Z(s)$. Closed-form solutions are obtained in a number of special cases. It is shown that for the case $m = 0$ there exists a third sequence of patterns (in addition to the Butterworth and Thomson functions) in which the second derivative of $A(\omega)$ vanishes at the origin, as do the first $(n - 2)$ even derivatives of $\Phi'(\omega)$.

I. INTRODUCTION

In connection with the design of cascaded amplifiers, it is of interest to inquire how the coefficients of the low-pass transfer function

$$Z(s) = \frac{P(s)}{Q(s)} = K\frac{1 + b_1 s + b_2 s^2 + \cdots + b_m s^m}{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n} (n > m), \quad (1a)$$

$$Z(i\omega) = A(\omega) \exp [i\Phi(\omega)] \quad (1b)$$

may be adjusted so as to yield a compromise between flat amplitude response and linear phase. For the purpose of this discussion, amplitude flatness will be achieved by making derivatives of $A(\omega)$ vanish at $(\omega = 0)$. Phase linearity will be obtained by imposing similar flatness conditions on the derivative of the phase function. The problem may then be stated as follows:

For a given degree of complexity (that is, for given m, n) determine the coefficients a_i and b_j so that the first r even derivatives of $A(\omega)$ and the first $(n + m - 1 - r)$ even derivatives of $d\Phi/d\omega$ vanish at the origin.

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This set of conditions will be referred to as r "amplitude conditions" and $(n + m - 1 - r)$ "phase conditions." The choice of r determines the relative emphasis on amplitude and phase, respectively. The fact that the derivatives are required to vanish at $\omega = 0$ implies a restriction to low-pass systems or their narrow-band band-pass analogs. In the above formulation, it is assumed implicitly that there exist no constraint relations among the a_i and b_j beyond those imposed by the fact that the coefficients are real.

Solutions of the problem are available in the literature for transfer functions consisting of poles only,[3] under the conditions of maximally flat amplitude $(r = n - 1)$ and maximally linear phase $(r = 0)$. Maximally flat amplitude functions are characterized by poles equispaced on a semicircle (Butterworth distribution (1).[4] The maximally linear phase sequence has been investigated by Laplume (2), Thomson (3), and Storch (4). Unlike the Butterworth case, the maximally linear phase sequence led to closed-form expressions for the coefficients of $Q(s)$; pole patterns had to be determined by numerical methods.

Peless and Murakami (5) recently published a detailed investigation of patterns consisting of poles only and designed to yield compromises between amplitude flatness and phase linearity. Their method is based on geometric interpolation between the Butterworth and Thomson pole patterns, and does not use the derivative approach outlined above. Derivative adjustment for transfer functions containing zeros has apparently not been treated in the literature, beyond the particular case of a shunt-peaking network which was considered by Lynch (6). (This case involves one pair of poles and one zero, with constraint relations between them.)

The present development reduces the general case to a set of simultaneous equations in the a_j and b_j. Closed-form solutions for the coefficients are obtained in a number of special cases. It is shown that in the case of transfer functions without zeros[5] there exists a third sequence of pole patterns which arises from the application of one amplitude and $(n - 2)$ phase conditions.

In Section II, certain relations between the coefficients of a polynomial and its roots are recalled. Section III presents a number of ways in which amplitude and phase conditions can be expressed; in particular, it is shown that these conditions can be stated quite simply in terms of sums of powers of the reciprocal zeros and poles. This formulation is used in Section IV in conjunction with Newton's Identities to derive the set of simultaneous equations for the general case. This result is then applied to a number of special cases.

[3] Zeros at infinity will not be considered when specifying the number of zeros of $Z(s)$.

[4] The boldface numbers in parentheses refer to the references appended to this paper.

[5] This case has been investigated extensively by M. J. E. Golay on the basis of continued-fraction expansions (8).

[6] This procedure is the basis of Storch's investigation (4).

II. NEWTON'S IDENTITIES

The following facts of algebra will be utilized in the subsequent derivations. The coefficients of a polynomial

$$Q(s) = 1 + a_1 s + a_2 s^2 + \cdots + a_n s^n \tag{2}$$

having roots $\alpha_1, \alpha_2, \cdots, \alpha_n$ and the functions

$$\sigma_k = \sum_{j=1}^{n} \left(\frac{1}{\alpha_j}\right)^k \tag{3}$$

can be related by Newton's Identities:

$$\begin{aligned}
\sigma_1 + a_1 &= 0 \\
\sigma_2 + a_1\sigma_1 + 2a_2 &= 0 \\
\sigma_3 + a_1\sigma_2 + a_2\sigma_1 + 3a_3 &= 0 \\
\sigma_4 + a_1\sigma_3 + a_2\sigma_2 + a_3\sigma_1 + 4a_4 &= 0 \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\
\sigma_k + a_1\sigma_{k-1} + a_2\sigma_{k-2} + \cdots + ka_k &= 0
\end{aligned} \tag{4}$$

where $a_i \equiv 0$, when $i > n$.

III. FORMULATION OF DERIVATIVE CONDITIONS

Lynch(6) has pointed out that consideration of the general case could be simplified by the observation that the first r derivatives of

$$A^2(\omega) = \left[\frac{P(s)P(-s)}{Q(s)Q(-s)}\right]_{s=i\omega} = K\left[\frac{1 + c_2 s^2 + c_4 s^4 + \cdots}{1 + d_2 s^2 + d_4 s^4 + \cdots}\right]_{s=i\omega} \tag{5a}$$

vanish at the origin if

$$c_{2k} = d_{2k} \qquad \text{for} \qquad k = 1, 2, \cdots, r. \tag{5b}$$

The restriction $m < n$ of (1a) implies that the number of amplitude conditions must be less than the number of poles. Of the $(n + m)$ coefficients of $Z(s)$, one may be taken as a scale factor: a_1 is used as such in this discussion. It follows that the total number of amplitude and phase conditions cannot exceed $(m + n - 1)$.

Using (5) as a starting point, amplitude conditions on the transfer function $Z(s)$ may be expressed in terms of the a_i and b_j. The first r derivatives of $A(\omega)$ vanish at the origin if

$$a_k^2 - 2\sum_{j=1}^{k} (-1)^{j+1} a_{k+j} a_{k-j} = b_k^2 - 2\sum_{j=1}^{k} (-1)^{j+1} b_{k+j} b_{k-j}$$

$$\text{for} \qquad k = 1, 2, \cdots, r. \tag{6}$$

While similar derivative conditions may be applied to $d\Phi/d\omega$, it is generally more convenient to impose phase conditions by expanding $\Phi(\omega)$ in the form (6)

$$\Phi(\omega) = \arctan\left[r_1\omega + r_3\omega^3 + r_5\omega^5 + \cdots\right], \tag{7a}$$

and by equating successive r's to the corresponding coefficients of the desired characteristic

$$\begin{aligned}\Phi_{\text{desired}}(\omega) &= r_1\omega = \arctan(\tan r_1\omega)\\ &= \arctan\left[r_1\omega + (1/3)(r_1\omega)^3 + (2/15)(r_1\omega)^5 + \cdots\right].\end{aligned} \tag{7b}$$

A similar procedure may be used in conjunction with the continued-fraction expansion[8] of $\Phi(\omega)$. With suitable normalization of the frequency variable, one may write

$$\Phi(\omega) = i\arctan\left\{1\Big]\frac{1}{s} + 1\Big]\frac{m_2}{s} + \cdots + 1\Big]\frac{m_{n-1}}{s} + 1\Big]\frac{m_n}{s}\right\}_{s=i\omega}. \tag{8a}$$

Successive m_k are then equated to the corresponding coefficients of the expansion

$$\begin{aligned}\Phi_{\text{desired}}(\omega) &= -\omega = i\operatorname{arctanh}\left[\tanh s\right]_{s=i\omega}\\ &= i\operatorname{arctanh}\left\{1\Big]\frac{1}{s} + 1\Big]\frac{3}{s} + 1\Big]\frac{5}{s}\cdots\right\}_{s=i\omega}\end{aligned} \tag{8b}$$

Finally, both amplitude and phase conditions may be expressed in terms of sums of powers of the poles and zeros of $Z(s)$. This formulation is essential to the present discussion. Let

$$\sigma_k \equiv \sum_{i=1}^{n}\left(\frac{1}{\alpha_i}\right)^k, \tag{9a}$$

$$\bar{\sigma}_k = \sum_{i=1}^{m}\left(\frac{1}{\beta_i}\right)^k, \tag{9b}$$

where the α_i and β_j are the poles and zeros of the transfer function $Z(s)$. It is shown in Appendix A that the first r even derivatives of $A(\omega)$ vanish at the origin if

$$\sigma_{2k} = \bar{\sigma}_{2k} \qquad \text{for } k = 1, 2, \cdots, r, \tag{10}$$

and that, independently of (10), the r^{th} even derivative of $d\Phi/d\omega$ vanishes at the origin if

$$\sigma_{2r+1} = \bar{\sigma}_{2r+1}. \tag{11}$$

IV. CALCULATION OF COEFFICIENTS

The general solution will be presented first, followed by a number of special cases.

A. General Solution

Newton's Identities may be written in the form

$$\begin{aligned}
\sigma_1+a_1&=\bar{\sigma}_1+b_1\\
\sigma_2+a_1\sigma_1+2a_2&=\bar{\sigma}_2+b_1\bar{\sigma}_1+2b_2\\
\sigma_3+a_1\sigma_2+a_2\sigma_1+3a_3&=\bar{\sigma}_3+b_1\bar{\sigma}_2+b_2\bar{\sigma}_1+3b_3\\
\sigma_4+a_1\sigma_3+a_2\sigma_2+a_3\sigma_1+4a_4&=\bar{\sigma}_4+b_1\bar{\sigma}_3+b_2\bar{\sigma}_2+b_3\bar{\sigma}_1+4b_4\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
\sigma_k+a_1\sigma_{k-1}+a_2\sigma_{k-2}+\cdots+ka_k&=\bar{\sigma}_k+b_1\bar{\sigma}_{k-1}+b_2\bar{\sigma}_{k-2}+\cdots+kb_k
\end{aligned} \tag{12}$$

where $a_i \equiv 0$ when $i > n$, and $b_j \equiv 0$ when $j > m$. The desired number of amplitude and phase conditions may now be introduced in the form $\sigma_2 = \bar{\sigma}_2$, $\sigma_3 = \bar{\sigma}_3$, etc., and the resulting set of equations must be solved for the a_i and b_j with the aid of (4). It should be pointed out that the procedure does not guarantee physical realizability; that is, solutions with right half-plane poles are obtained in some cases.

A convenient starting point for the consideration of special cases is the situation in which the number of amplitude conditions equals the number of phase conditions or exceeds it by one, according as $(n + m)$ is odd or even: this choice places roughly equal emphasis on amplitude flatness and phase linearity and will therefore be designated as the *equal-emphasis* case.

B. The Equal-Emphasis Case

The significant feature of this case is the fact that $\sigma_j = \bar{\sigma}_j$ for *all* successive values of j between $2 \leqq j \leqq (n + m - 1)$. Substitution of these relations into (12) and rearrangement of terms yields

$$\begin{aligned}
2(a_2-b_2)&=a_1^2-b_1^2\\
3(a_3-b_3)&=-(a_1-b_1)\bar{\sigma}_2+(a_1a_2-b_1b_2)\\
4(a_4-b_4)&=-(a_1-b_1)\bar{\sigma}_3-(a_2-b_2)\bar{\sigma}_2+(a_1a_3-b_1b_3)\\
5(a_5-b_5)&=-(a_1-b_1)\bar{\sigma}_4-(a_2-b_2)\bar{\sigma}_3-(a_3-b_3)\bar{\sigma}_2+(a_1a_4-b_1b_4)\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots
\end{aligned} \tag{13}$$

Using (4), the right-hand sides of these equations can be expressed in terms of a_1 and the b_j only. One obtains

$$a_2-b_2=\frac{1}{2!}(a_1-b_1)(a_1+b_1)$$

$$a_3-b_3=\frac{1}{3!}(a_1-b_1)^2(a_1+2b_1)+(a_1-b_1)b_2$$

$$a_4-b_4=\frac{1}{4!}(a_1-b_1)^3(a_1+3b_1)+\frac{1}{2}(a_1-b_1)^2b_2+(a_1-b_1)b_3 \tag{14}$$

$$a_k-b_k=\frac{1}{k!}(a_1-b_1)^{k-1}[a_1+(k-1)b_1]+\frac{1}{(k-2)!}(a_1-b_1)^{k-2}b_2$$

$$+\frac{1}{(k-3)!}(a_1-b_1)^{k-3}b_3+\cdots+(a_1-b_1)b_{k-1}, \qquad (k>2);$$

where $a_k \equiv 0$ if $k > n$ and $b_k \equiv 0$ if $k > m$.

Explicit expressions are given below for $m = 0, 1, 2$.

(i) No zeros ($m = 0$):

$$a_k = \frac{1}{k!}a_1^k, \tag{15}$$

(ii) One zero ($m = 1$):

$$b_1 = -\frac{a_1}{n} \tag{16a}$$

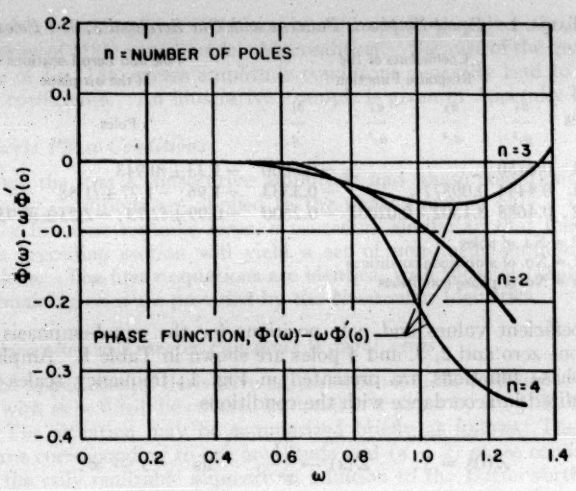
$$a_k = \frac{(n+1)^{k-1}(n-k+1)}{n^k k!}a_1^k \tag{16b}$$

(iii) Two zeros ($m = 2$):

$$b_1 = -\frac{2a_1}{n}; \qquad b_2 = \frac{n+2}{n^2(n+1)}a_1^2, \tag{17a}$$

$$a_k = \frac{(n+2)^{k-1}}{n^k k!}\left[(n-2k+2)+\frac{k(k-1)}{n+1}\right]a_1^k. \tag{17b}$$

The realizability of these three cases was investigated by applying the Hurwitz criteria (7). Case (i) leads to physically unrealizable pole patterns for $n \geqq 5$, and (ii) ceases to be realizable when $n = 7$. The upper limit for the realizability of (iii) has not been determined exactly, but is known to lie between $n = 8$ and $n = 40$. It is seen that cases (ii) and (iii) have their zeros in the right half-plane, a result which might have been expected in view of the simultaneous conditions on amplitude and phase.



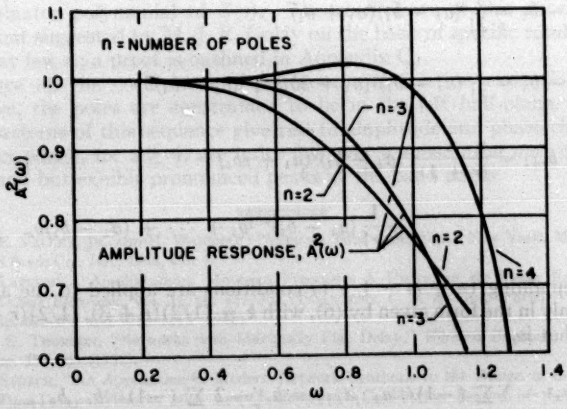


FIG. 1. Frequency characteristics of equal-emphasis functions with one zero.

TABLE I.—*Equal-Emphasis Patterns with One Zero and 2, 3, 4 Poles.*

			Coefficients of the Response Function				Pole and Zero Locations in the $a_1 s$-plane	
P*	A	Ph	$\frac{a_2}{a_1^2}$	$\frac{a_3}{a_1^3}$	$\frac{a_4}{a_1^4}$	$\frac{b_1}{a_1}$	Poles	Zero
2	1	1	0.3750			−0.5000	$-1.33 \pm i0.943$	2.00
3	2	1	0.4444	0.09877		−0.3333	-1.96; $-1.27 \pm i1.88$	3.00
4	2	2	0.4688	0.1302	0.02035	−0.2500	$-0.99 \pm i2.73$; $-2.23 \pm i0.93$	4.00

* P = No. of poles
A = No. of amplitude conditions
Ph = No. of phase conditions

Coefficient values and pole positions for the equal-emphasis case with one zero and 2, 3, and 4 poles are shown in Table I. Amplitude and phase functions are presented in Fig. 1; frequency scales were normalized in accordance with the conditions

$$Z(0) = 1; \qquad Z(s) \to \frac{1}{s^{n-1}}, \qquad \text{as} \qquad s \to \infty.$$

C. Excess Amplitude Conditions

The first r conditions ($r < n + m - 1$) are imposed alternately on amplitude and phase, so that the first r equations of (14) apply:

$$a_2 - b_2 = \frac{1}{2!}(a_1 - b_1)(a_1 + b_1)$$

$$a_3 - b_3 = \frac{1}{3!}(a_1 - b_1)^2(a_1 + 2b_1) + (a_1 - b_1)b_2 \tag{18a}$$

$$a_{r+1} - b_{r+1} = \frac{1}{(r+1)!}(a_1 - b_1)^r(a_1 + rb_1) + \frac{1}{r!}(a_1 - b_1)^{r-1}b_2 + \cdots + (a_1 - b_1)b_r. \tag{18a}$$

The remaining $(n + m - 1 - r)$ conditions are applied to the amplitude only in the form given by (6), with $k = (1/2)(r + 3), (1/2)(r + 5)$, etc.; that is,

$$a_k^2 - 2\sum_{j=1}^{k}(-1)^{j+1}a_{k-j}a_{k+j} = b_k^2 - 2\sum_{j=1}^{k}(-1)^{j+1}b_{k-j}b_{k+j}. \tag{18b}$$

The last step is simplified if the number of zeros $m \leq r$, since the right-hand sides of (18*b*) vanish under this condition. Because of the quadric nature of Eq. 18*b*, excess amplitude conditions generally lead to irrational coefficients. An illustrative example is given in Appendix B.

D. Excess Phase Conditions

After the first r consecutive amplitude and phase conditions, the remaining conditions are applied to the phase function only. If the transfer function contains zeros, a procedure similar to that followed in the preceding section will yield a set of non-linear simultaneous equations. The first r equations are identical with those of (18*a*), and the remaining ones are provided by the Newtonian Identities.

E. Application to Transfer Functions Without Zeros

The application of derivative-adjustment methods to transfer functions with $m = 0$ will be covered in detail in a paper by M. J. E. Golay (8). The situation may be summarized briefly as follows: The pole patterns corresponding to one amplitude and $(n - 2)$ phase conditions form the only realizable sequence in addition to the Butterworth and Thomson sequences. The new sequence is determined by the partial fraction expansion

$$\frac{N_n(p)}{M_n(p)} = 1\Big|\frac{1}{p} + 1\Big|\frac{3}{p} + 1\Big|\frac{5}{p} + \cdots + 1\Big|\frac{2n-3}{p} + 1\Big|\frac{2}{p} \qquad (19)$$

where $p = a_1 s$, and $M_n(p)$ and $N_n(p)$ are the even and odd parts of the denominator polynomial of $Z(s)$. The coefficients of this expansion were first suggested by M. J. E. Golay on the basis of specific results for the first few n; a proof is outlined in Appendix C.

Since all the coefficients of the continued-fraction expansion are positive, the poles are constrained to lie in the left half-plane. The pole patterns of this sequence give rise to amplitude and phase characteristics which, for $n \geq 4$, are quite flat over a substantial part of the passband but exhibit pronounced peaks at the band edges.

REFERENCES

(1) G. E. VALLEY, JR. AND H. WALLMAN: "Vacuum Tube Amplifiers," New York, McGraw-Hill Book Co., Inc., 1948, Ch. 4.

(2) J. LAPLUME, "Amplificateurs Moyenne Frequence à Distorsion de Phase Reduite," *L'Onde Electrique*, Vol. 31, pp. 357–362 (1951); also, *C. R. Ac. Sc.*, (Paris), Vol. 227, pp. 675–677 and 1213–1215 (1948).

(3) W. E. THOMSON, "Networks with Maximally Flat Delay," *Wireless Engineer*, Vol. 29, pp. 256–263 (1952).

(4) L. STORCH, "An Application of Modern Network Synthesis to the Design of Constant-Time-Delay Networks with Low-Q Elements," Convention Record of the I.R.E., Part 2, pp. 105–117, 1954.

(5) Y. PELESS AND T. MURAKAMI, "Analysis and Synthesis of Transitional Butterworth-Thomson Filters and Bandpass Amplifiers," *RCA Review*, Vol. 18, pp. 60–94 (1957).
(6) W. A. LYNCH, "The Role Played by Derivative Adjustment in Broadband Amplifier Design," Proc. Symp. on Modern Network Synthesis, Polytechnic Institute of Brooklyn, 1952, pp. 193–201.
(7) E. A. GUILLEMIN: "The Mathematics of Circuit Analysis," New York, John Wiley & Sons, 1949, p. 395.
(8) M. J. E. GOLAY: "Lumped Constant Networks Satisfying Conditions at the Origin," (To be published.)

APPENDIX A

Proof of Eqs. 10 and 11

The s^2-roots of the polynomial $P(s)P(-s)$ appearing in (5*a*) are the squares of the zeros of $Z(s)$; similarly, the s^2-roots of $Q(s)Q(-s)$ are the squares of the poles of $Z(s)$. Newton's Identities for the two polynomials in s^2 of (5*a*) are

$$\begin{aligned} \sigma_2 + c_2 &= 0 & \bar{\sigma}_2 + d_2 &= 0 \\ \sigma_4 + c_2\sigma_2 + 2c_4 &= 0 & \bar{\sigma}_4 + d_2\bar{\sigma}_2 + 2d_4 &= 0 \\ \sigma_6 + c_2\sigma_4 + c_4\sigma_2 + 3c_6 &= 0 & \bar{\sigma}_6 + d_2\bar{\sigma}_4 + d_4\bar{\sigma}_2 + 3d_6 &= 0 \\ \cdots\cdots\cdots\cdots & & \cdots\cdots\cdots\cdots & \\ \sigma_{2k} + c_2\sigma_{2k-2} + \cdots + kc_{2k} &= 0 & \bar{\sigma}_{2k} + d_2\bar{\sigma}_{2k-2} + \cdots + kd_{2k} &= 0 \end{aligned} \tag{A-1}$$

where σ_i and $\bar{\sigma}_i$ are defined by (9). Applying the condition of (5*b*) to (A-1), one obtains (10)

The phase of the transfer function $Z(s)$ is

$$\Phi(\omega) = \sum_{j=1}^{m} \arctan\left(\frac{\omega}{-\beta_j}\right) - \sum_{j=1}^{n} \arctan\left(\frac{\omega}{-\alpha_j}\right). \tag{A-2}$$

and

$$\frac{d\Phi}{d\omega} = \sum_{j=1}^{n} \frac{\alpha_j}{\alpha_j^2 - (i\omega)^2} - \sum_{j=1}^{m} \frac{\beta_j}{\beta_j^2 - (i\omega)^2}.$$

Expanding each term about the origin,

$$\frac{d\Phi}{d\omega} = \sum_{j=1}^{n} \sum_{k=0}^{\infty} \frac{(i\omega)^{2k}}{\alpha_j^{2k+1}} - \sum_{j=1}^{m} \sum_{k=0}^{\infty} \frac{(i\omega)^{2k}}{\beta_j^{2k+1}}. \tag{A-3}$$

Interchanging orders of summation, and applying the definition of (9), one obtains

$$\frac{d\Phi}{d\omega} = \sum_{k=0}^{\infty} (\sigma_{2k+1} - \bar{\sigma}_{2k+1})(i\omega)^{2k}, \tag{A-4}$$

and (11) follows.

APPENDIX B

Illustrative Example

Excess Amplitude Conditions

The pattern to be considered is one consisting of four poles and one zero ($n = 4$, $m = 1$), arranged in accordance with 3 amplitude and one phase condition. In this case, $r = 3$, and (14) provides the following equations:

$$\begin{aligned} a_2 &= (1/2)(a_1^2 - b_1^2) \\ a_3 &= (1/6)(a_1 - b_1)^2(a_1 + 2b_1) \\ a_4 &= (1/24)(a_1 - b_1)^3(a_1 + 3b_1). \end{aligned}$$

The remaining equation is obtained from (18*b*) with $k = 3$:

$$a_3^2 = 2a_2a_4.$$

One obtains

$$b_1 = (-2 + \sqrt{3})a_1$$
$$a_2 = (-3 + 2\sqrt{3})a_1^2$$
$$a_3 = (-12 + 7\sqrt{3})a_1^3$$
$$a_4 = \tfrac{1}{3}(-45 + 26\sqrt{3})a_1^4.$$

APPENDIX C

Proof of Eq. 19

With $p = a_1 s$, the transfer function $Z(s)$ may be written in the form

$$Z(s) = \frac{K'}{A_n + A_n p + B_n p^2 + \cdots + p^n} = \frac{K'}{M_n(p) + N_n(p)}. \tag{C-1}$$

Imposing $(n-2)$ phase conditions consists of making the first $(n-1)$ coefficients of the continued-fraction expansion of $N_n(p)/M_n(p)$ equal to the corresponding coefficients of $\tanh(p)$; that is,

$$\frac{N_n(p)}{M_n(p)} = \underline{1}\rfloor\frac{1}{p} + \underline{1}\rfloor\frac{3}{p} + \underline{1}\rfloor\frac{5}{p} + \underline{1}\rfloor\cdots + \underline{1}\rfloor\frac{2n-3}{p} + \underline{1}\rfloor\frac{m_n}{p}. \tag{C-2}$$

The last coefficient, m_n, must be determined by use of the amplitude condition

$$A_n - 2B_n = 0. \tag{C-3}$$

A_n and B_n may be expressed in terms of m_n; for $n \geq 2$

$$A_n = x_n m_n, \qquad B_n = y_n + z_n m_n;$$

where x, y, and z satisfy the recurrence relations

$$x_{n+1} = (2n-1)x_n,$$
$$y_{n+1} = (2n-3)y_n,$$
$$z_{n+1} = (2n-1)z_n + y_n;$$

with $x_2 = y_2 = 1$, $z_2 = 0$. The amplitude condition (C-3) may now be written as

$$(x_n - 2z_n)m_n - 2y_n = 0,$$

and one may show by induction that $m_n = 2$; that is, that the equation

$$x_n - 2y_n - z_n = 0$$

is satisfied for all $n \geq 3$.

CONCERNING THERMAL JUNCTION RESISTANCE IN THE A. F. JOFFE METHOD FOR MEASUREMENT OF THERMAL CONDUCTIVITY

BY

W. F. G. SWANN [1]

INTRODUCTION

In a recent paper[2] the writer has discussed the theory of the Joffe method for measurement of thermal conductivity. In a note at the end of the paper an expression was given showing that thermal junction resistance at the hot end of the specimen did not affect the determination of the quantity λ_0 which occurs in the exponent of e in the exponential decay of temperature with time, although, of course, the relation between λ_0 and the thermal conductivity does involve the junction resistance. Since considerable interest has been expressed in the aforesaid matter, it has been thought well to give the formal derivation of the aforesaid expression. At the same time, opportunity will be taken to correct a slip in sign[3] in the original paper, which correction does not, however, affect the basic developments in the paper. Reference to the original paper should be made for the meaning of the notation.

CORRIGENDA

We shall first cite the correction aforesaid. It is not involved at all in the main part of the paper which is not concerned with thermal junction resistance.

The correction appears in Eq. 25, where the sign of the term involving h should be reversed. The same remark applies to all *later* equations *containing h*, except for the equation on p. 380. The same correction applies in the abstract. The effect of this correction is to increase the measured value for the thermal conductivity. A few minor and unimportant changes in wording to correspond to the above change in sign will be obvious.

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[1] Director Emeritus of the Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa., and a Senior Staff Advisor for The Franklin Institute Laboratories for Research and Development.

[2] W. F. G. Swann, JOUR. FRANKLIN INST., Vol. 267, pp. 363–380 (1959).

[3] This slip in sign was kindly pointed out to the writer by Dr. J. S. Thomsen of the Johns Hopkins University, Mechanical Engineering Department.

FORMAL DEVELOPMENT OF EXPRESSION ON PAGE 380

In accordance with the development in the main paper we have, at the junction of B and S

$$\theta = \theta_0 e^{-\lambda_0 t}.$$

Here θ is the temperature of the *specimen* at the junction. The temperature θ_B of the *block* B at the junction is given, in accordance with footnote (3), by

$$Ms\frac{d}{dt}(\theta + \Delta\theta) = Ms\frac{d\theta_B}{dt} = -h(\theta_B - \theta)$$

$$\frac{Ms}{h}\frac{d\theta_B}{dt} + \theta_B = \theta = \theta_0 e^{-\lambda_0 t}$$

$$\frac{d\theta_B}{dt} + \frac{h}{Ms}\theta_B = \frac{\theta_0 h}{Ms}e^{-\lambda_0 t}$$

$$e^{ht/Ms}\theta_B = \frac{\theta_0 h}{Ms}\int [e^{ht/Ms}e^{-\lambda_0 t}] + c$$

where c is a constant to be determined so that $\theta_B = \theta_0$ at $t = 0$. The solution is

$$\theta_B = \frac{\theta_0 e^{-\lambda_0 t}}{1 - \lambda_0 Ms/h}\left[1 - \frac{\lambda_0 Ms}{h}e^{-ht/Ms(1-\lambda_0 Ms/h)}\right] \qquad (A)$$

Now, in general, it is legitimate to neglect the second term in the square bracket. For referring to the third equation (corrected) on p. 374, we see that the absolute value, δk, of the error in k resulting from neglect of thermal resistance is given by

$$\delta k = \left(\frac{k\lambda_0}{hl}\right)(Msl).$$

Therefore

$$\frac{\delta k}{k} = \frac{\lambda_0 Ms}{h}.$$

Let us write ϵ for $\delta k/k$. Then Eq. A above becomes

$$\theta_B = \frac{\theta_0 e^{-\lambda_0 t}}{1 - \lambda_0 Ms/h}[1 - \epsilon e^{-\lambda_0(1-\epsilon)t/\epsilon}] \qquad (B)$$

$$-\left(\frac{1}{\theta_B}\right)\frac{d\theta_B}{dt} = \lambda_0 - \frac{d}{dt}\log[1 - \epsilon e^{-\lambda_0(1-\epsilon)t/\epsilon}]$$

$$= \lambda_0\left[1 - \frac{(1-\epsilon)}{e^{\lambda_0(1-\epsilon)t/\epsilon} - \epsilon}\right]. \qquad (C)$$

Let us consider a case where $\delta k/k(=\epsilon)$ is less than 0.1, and suppose we do not take observations until a time $t = \tau$. Then, at this time the second term in the square bracket of (C) amounts to $-0.9/(e^{9\lambda_0\tau} - 0.1)$.

Now suppose the time τ for which we start to use the observations is such that $e^{-\lambda_0\tau} = 2/3$, that is, such that θ has fallen from θ_0 to $(2/3)\theta_0$. Then $e^{9\lambda_0\tau} = (3/2)^9 \sim 38$, and the second term in the square bracket amounts to only 0.025. For such a case, as far as experimental determination of λ_0 is concerned, we can write

$$-\left(\frac{1}{\theta_B}\right)\frac{d\theta_B}{dt} = \lambda_0$$

and indeed (A) approximates to the expression given on p. 380 of the original paper.

Of course, if higher accuracy is desired we can always use (C) as it stands, calculating an approximate value of λ_0 by neglecting the term involving ϵ, and then substituting this value of λ_0 in the said term for the purpose of calculating a more exact value of λ_0.

STABILITY AND OTHER RESULTS OF THE ROTATING MACHINE OSCILLATOR

BY

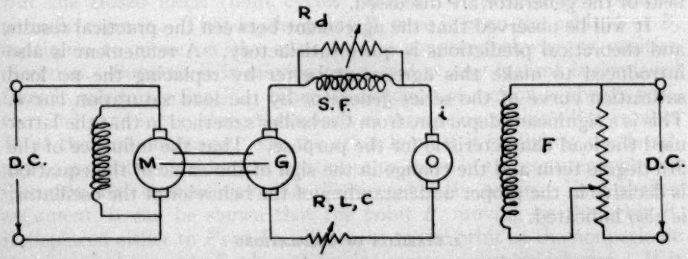
C. V. GOVINDA RAO [1]

ABSTRACT

A separately excited d-c. motor supplied from a d-c. series generator performs an unusual type of oscillation. For a reasonably clear understanding of the electro-mechanical phenomenon underlying this behavior, a nonlinear differential equation was developed and its complete solution was obtained by partially graphical and numerical methods. In particular Liénard's method was adopted to examine the possibility of any periodic solutions. In this method periodic solutions are easily recognized by the presence of the so called "limit cycles," their number being equal to the number of such loops. Of these, one or more may give stable solutions and the rest unstable. In a set of investigations with which this paper is concerned two limit cycles in general were observed of which one and only one represents stable oscillations. The existence of these limit cycles itself was found to depend on various factors such as the polar movement of inertia, the shaft load, the brush shift, the circuit resistance and excitation. The paper confines itself to the discussion of the effects of variation of the first three factors and in the main the stability of oscillations. Any additional facts observed are also recorded.

1. INTRODUCTION

It is known that when a d-c. series generator feeds a separately excited d-c. motor, the latter performs a curious type of oscillations under certain conditions. Essential connections of such a combination are shown in Fig. 1. The motor M starts from rest, accelerates to reach



$$e_g = \alpha i + (\beta/3)i^3 - (r/5)i^5$$

$$\alpha = 40.74, \quad \beta = 3.53, \quad r = -0.69$$

FIG. 1. Rotating machine oscillator.

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a definite speed, then slows down to rest and restarts in the reverse direction to repeat the phenomenon all over again. If the rotational movement of inertia is comparatively small, the motor makes several revolutions between start in one direction and start in the other. If it is relatively large, that is, nearly equal to the moment of inertia of the supplying generator, complete revolutions may not take place at all. Then mechanical oscillations very similar to that of the Kapp's vibrator will result but with very low frequencies. In either case the instantaneous rotor velocity varies nonlinearly giving rise to electrical oscillations of relaxation type. To explain this somewhat paradoxical behavior of the oscillator, a nonlinear differential equation was developed whose solution was given by a semigraphical–seminumerical method based on Liénard's construction in an earlier paper (**2**).[2] There several conclusions were drawn which required experimental verification. But from experiments conducted for the purpose, some of which will be described in the later sections of the paper, it was found that the measured values of circuit current were at variance with the values determined by the solution indicated in (**2**). A re-examination of stability of oscillations revealed that while it was essential that the circuit resistance should be made less than its critical resistance for building up oscillations, there are in general two distinct regimes of periodic oscillations one of which is unstable and the other stable. Liénard's construction clearly indicated these two possible regimes by the existence of two closed loops called limit cycles and the limit cycle given in the previous paper corresponded to the unstable regime. Section 1 is devoted to a general discussion of stability of oscillations. In Sections 3, 4, 5 and 6 effects of increased polar movement of inertia, shaft load, brush shift and introduction of a diverter across the series field of the generator are discussed.

It will be observed that the agreement between the practical results and theoretical predictions is quite satisfactory. A refinement is also introduced to make this agreement better by replacing the no load saturation curve of the series generator by the load saturation curve. This is a significant departure from Corbeiller's method in that the latter used the load characteristic for the purpose. That the influence of the 5th degree term and the change in the sign of the cubic of the equation is decisive in the proper understanding of the behavior of the oscillator, is also indicated.

2. STABILITY OF OSCILLATIONS

The electrodynamical equation governing the performance of the oscillator shown in Fig. 1 may be written down from inspection as

$$L\frac{di}{dt} + Ri + \frac{K\phi^2}{J}\int idt = \alpha i + \frac{\beta}{3}i^3 - \frac{\gamma}{5}i^5 \qquad (1)$$

[2] The boldface numbers in parentheses refer to the references appended to this paper.

where L, R, K, α, β and γ are parameters determined for the series circuit from experiments. The right hand side expression gives the generated voltage of the supplying series machine taking saturation into account. Putting $\int idt = q$, $n^2 = \frac{K\phi^2}{L} = \frac{K_1\phi^2}{J}$, $K\phi^2 = \frac{1}{C}$ and $\frac{R-\alpha}{nL} = \mu$, the equation can at once be reduced to the form

$$\ddot{q} + \mu\{(\dot{q} - a\dot{q}^3 + b\dot{q}^5)\} + q = 0 \tag{2}$$

from which we further may have

$$q = -\mu(\dot{q} - a\dot{q}^3 + b\dot{q}^5). \tag{3}$$

The relationship between q and $\dot{q}$ for known values of μ, a and b which is often called the characteristic, may be plotted from Eq. 3. One such characteristic curve for estimated values of $\mu = 0.131$, $a = 1.2$ and $b = 0.109$ is shown in Fig. 2 marked C. The closed loops A and B graphically constructed as explained in (2) show the possible periodic regimes of oscillations. One limit cycle surrounds the inner humps and the other the outer humps (completely ignored in the previous paper). Of these A stands for the unstable regime and B for the stable regime.

Refer to Fig. 2. Consider a point P on one of the solution curves. Let Δq be the small increase of charge at the point considered and $\Delta q/\Delta t$ be the corresponding current. From the position of P it is seen that the current is positive and therefore the point P moves in the counterclockwise direction as time passes. This is true for all positions of P in the phase-plane plot.

The solution curves give periodic as well as nonperiodic solutions. But the closed loops (limit cycles) A and B represent two possible periodic regimes. That is, the point P, once it is on the orbit A or B, continues to move on the same in the counterclockwise or positive direction. Suppose it is moving on A and at any instant is displaced from it, say to a point P_1. Since its direction of motion is always positive, it has to move along the spirals of increasing radius till it reaches B, as the time increases. On the other hand if it is slightly displaced, say to a point P_2 inside A, it continues to move in the counterclockwise direction with decreasing radius till it reaches the origin. By a similar argument, it can be shown that the point P, moving on the orbit B, if displaced either to P_3 or P_4 will return to the orbit as the nonperiodic curves spiral towards B when the point moves counterclockwise. It is therefore evident that while P, if slightly displaced from the periodic regime corresponding to A, either moves into the origin or away from it till it reaches B, any displacement given to it from the orbit B, will always bring it back to B with the increase of time. Thus, of the two periodic regimes, that which corresponds to A is unstable and that which

corresponds to B is stable. The above discussion is true for all values of $R > \alpha$. For $R = \alpha$, the limit cycle A degenerates to a point coincident with the origin, and for $R < \alpha$, it does not exist at all, in which case the origin becomes unstable. The oscillations then build up.

A study of the characteristic curves constructed for various values of μ revealed, that the inner humps become more and more flat against the axis of current with the corresponding decrease of intercepts on it. For $\alpha = R$, these humps totally vanish which evidently indicates that

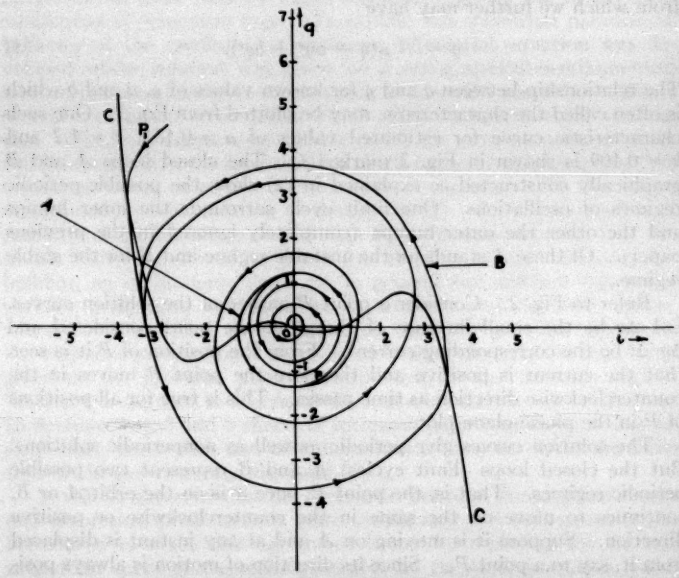


FIG. 2. (*a*) Closed loops showing two periodic regimes.

$\mu = 0.131$
$a = 1.2$
$b = 0.109$

the limit cycles corresponding to them do not exist. For the critical case $R = \alpha$, the oscillations could just build up if the oscillator is slightly disturbed from its position of rest or crawling. All experiments conducted in this connection proved that it is practically impossible to make the oscillator settle down to oscillations corresponding to limit cycle A; the motor invariably bursts into oscillations from the position of rest or state of crawling and never gradually and smoothly. For

$\mu = 0.131$ the limit cycle B gave an amplitude of 3.6 amp. whereas the experimental value was 3.9 amp. The current corresponding to the inner limit cycle was 1.1 amp. which could not sustain oscillations at all.

The discrepancy between the theoretical results and the measured quantities given above can further be reduced by taking the effect of armature reaction into consideration. This is best done by replacing the no-load magnetization curve, by the load magnetization curve and

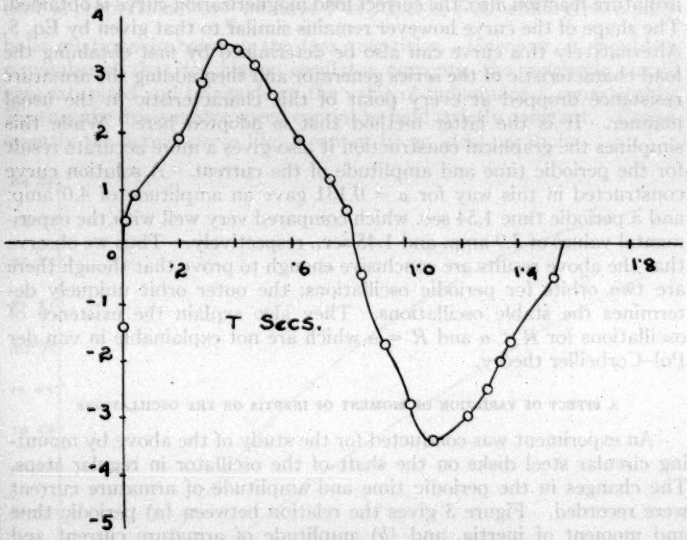


FIG. 2. (*b*) Current wave corresponding to closed loop.

deducing the characteristic curve C directly from this curve. For, the performance Eq. 1 after rearranging the terms can be written down as

$$L\frac{di}{dt} + (R - \alpha)i - \beta/3\, i^3 + \gamma/5\, i^5 + 1/C \int idt = 0.$$

This equation and Eq. 2 are identical except for the simple substitutions made. Therefore, the characteristic Eq. 3 is also given by

$$\frac{1}{CL}\int idt = -\frac{1}{L}\{(R - \alpha)i - \beta/3\, i^3 + \gamma/5\, i^5\} \qquad (4)$$

but for the change in the scale for the charge q. Rearranging the terms on the right hand side of Eq. 4 we have

$$\frac{1}{C}\int idt = \{\alpha i + \beta/3\, i^3 - \gamma/5\, i^5\} - Ri \qquad (5)$$

which could at once be recognized as the load characteristic when the armature reaction is neglected. If from this we deduct the effect of armature reaction also, the correct load magnetization curve is obtained. The shape of the curve however remains similar to that given by Eq. 5. Alternatively this curve can also be determined by first obtaining the load characteristic of the series generator and then adding the armature resistance dropped at every point of this characteristic in the usual manner. It is the latter method that is adopted here. While this simplifies the graphical construction it also gives a more accurate result for the periodic time and amplitude of the current. A solution curve constructed in this way for $\mu = 0.131$ gave an amplitude of 4.0 amp. and a periodic time 1.54 sec. which compared very well with the experimental values of 3.9 amp. and 1.45 sec., respectively. Thus we observe that the above results are conclusive enough to prove that though there are two orbits for periodic oscillations, the outer orbit uniquely determines the stable oscillations. They also explain the existence of oscillations for $R < \alpha$ and $R = \alpha$ which are not explainable in van der Pol–Corbeiller theory.

3. EFFECT OF VARIATION OF MOMENT OF INERTIA ON THE OSCILLATIONS

An experiment was conducted for the study of the above by mounting circular steel disks on the shaft of the oscillator in regular steps. The changes in the periodic time and amplitude of armature current were recorded. Figure 3 gives the relation between (*a*) periodic time and moment of inertia, and (*b*) amplitude of armature current and moment of inertia at a constant exciting current of the oscillator. It is noticed from curve (1) that the period of oscillations increases as the moment of inertia of the rotor is increased. That this should be so, may be deduced from the theoretical considerations also. For, we know from the substitutions indicated above that

$$\mu = (R - \alpha)\sqrt{\frac{C}{L}}. \qquad (6)$$

and

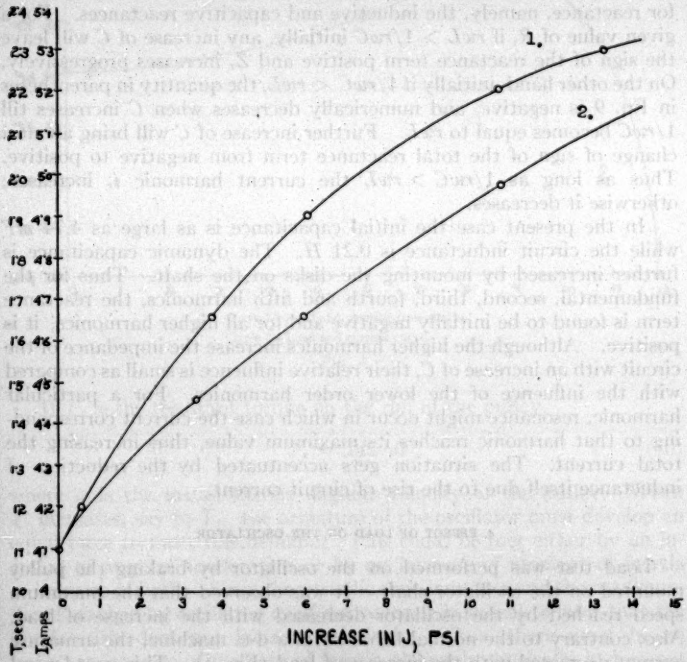
$$C = \frac{J}{K_1\phi^2}.$$

Any increase of J must result in the corresponding increase of μ and an increase of μ must affect the armature current wave in such a way as to

make it tend towards relaxation oscillations. As a consequence, the periodic time must increase. Squaring both sides of Eq. 6, we obtain

$$\mu^2 = (R - \alpha)^2 \frac{C}{L} = \frac{(R - \alpha)^2}{n^2 L^2} \tag{7}$$

from which we have $\frac{1}{n^2} = CL$. Since L is held constant and periodic time is proportional to $\frac{1}{n}$ we have $T^2 \alpha C$, that is, $T^2 \alpha J$ which gives a parabolic relationship between the two quantities. However, it should be remembered that for higher oscillator currents, the magnetic circuit gets saturated and brings down the value of inductance L considerably. Further the drive-motor speed cannot be held strictly constant. Consequently the actual curve Fig. 2 cannot be an ideal parabola.



Curve 1—Relation between I and J.
Curve 2—Relation between T and J.

FIG. 3.

It is also observed from Curve 2, Fig. 3 that the armature current increases as the moment of inertia is increased. This phenomenon could also be justified as follows:

The generator terminal voltage may be resolved into Fourier components and each component voltage may be considered to produce its own component current in the series circuit. Then, for any voltage harmonic e_r the current in the circuit is given by

$$i_r = e_r/Z_r = e_r/\{R + j(rwL - 1/rwC)\} \tag{8}$$

where Z_r is the "electrodynamic impedance" and w the fundamental angular frequency. Taking Z_r separately, we have

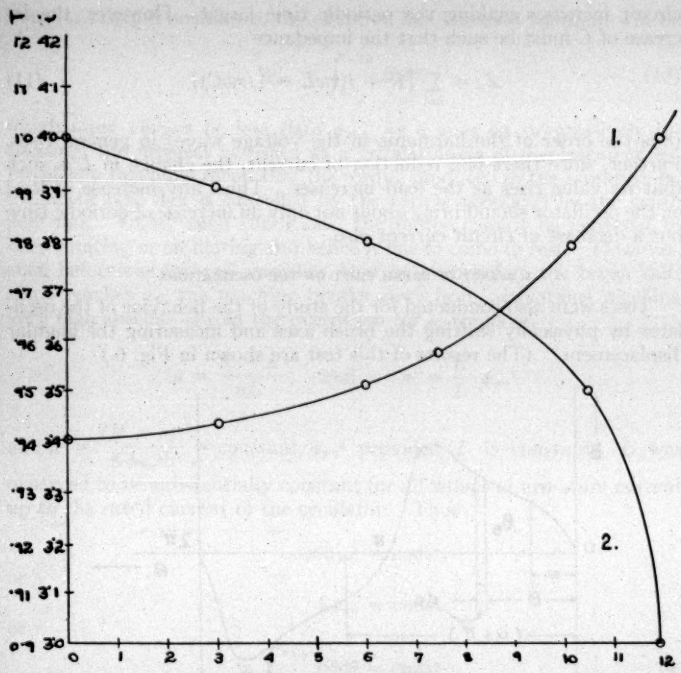
$$Z_r = R + j(rwL - 1/rwC) \tag{9}$$

in which the quantity within the parentheses gives the two components for reactance, namely, the inductive and capacitive reactances. For a given value of R, if $rwL > 1/rwC$ initially, any increase of C will leave the sign of the reactance term positive and Z_r increases progressively. On the other hand, initially if $1/rwC < rwL$, the quantity in parentheses in Eq. 9 is negative, and numerically decreases when C increases till $1/rwC$ becomes equal to rwL. Further increase of C will bring about a change of sign of the total reactance term from negative to positive. Thus as long as $1/rwC > rwL$ the current harmonic i_r increases; otherwise it decreases.

In the present case the initial capacitance is as large as 4.74 mF while the circuit inductance is 0.21 H. The dynamic capacitance is further increased by mounting the disks on the shaft. Thus for the fundamental, second, third, fourth and fifth harmonics, the reactance term is found to be initially negative and for all higher harmonics, it is positive. Although the higher harmonics increase the impedance of the circuit with an increase of C, their relative influence is small as compared with the influence of the lower order harmonics. For a particular harmonic, resonance might occur in which case the current corresponding to that harmonic reaches its maximum value, thus increasing the total current. The situation gets accentuated by the reduction of inductance itself due to the rise of circuit current.

4. EFFECT OF LOAD ON THE OSCILLATOR

Load test was performed on the oscillator by braking the pulley mounted on the oscillator shaft. It was observed that the maximum speed reached by the oscillator decreased with the increase of load. Also, contrary to the normal behavior of a d-c. machine, the armature current decreased with the increase of load (Fig. 4). This paradoxical behavior may be explained as follows:



Curve 1—Load and periodic time curve.
Curve 2—Load and current curve.

FIG. 4.

If T_r is the resultant torque on the rim of the pulley,

$$T_r = Jd\omega/dt \tag{10}$$

where ω is the instantaneous angular velocity of the pulley. When T_r increases, say to T_{r1}, the armature of the oscillator must develop an equal force to meet this demand. This could be met either by an increase of angular acceleration or by an increase of moment of inertia or increase of both. Since the periodic time increases and the maximum speed reached by oscillator itself decreases, it is logical to infer that the acceleration does not increase relatively. Thus, there must be an apparent increase of moment of inertia although physically it does not increase. This is equivalent to saying that the capacitance of the

circuit increases making the periodic time larger. However, the increase of C must be such that the impedance

$$Z_s = \sum_{r=1} \{R + j(rwL - 1/rwC)\} \tag{11}$$

(r is the order of the harmonic in the voltage wave) in general rises. Further, since there is a reduction of current, the change in L is such that its value rises as the load increases. Thus, any increase of load on the oscillator should bring about not only an increase of periodic time but a decrease of circuit current also.

5. EFFECT OF BRUSH SHIFT ON THE OSCILLATIONS

Tests were also conducted for the study of the behavior of the oscillator by physically shifting the brush axes and measuring the angular displacement. (The results of this test are shown in Fig. 6.)

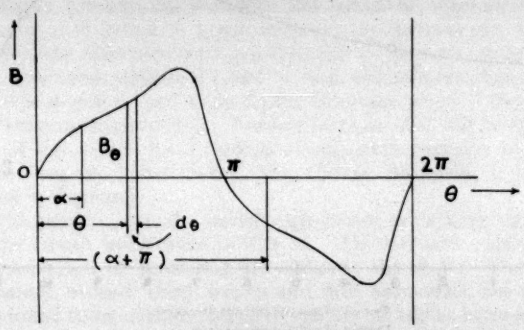


FIG. 5. Flux distribution in the air gap.

Figure 5 represents the approximate flux distribution in the air gap of a two-pole oscillator under load. For the maximum generated voltage, the brushes must be located on the magnetic neutral axis. On no load, this axis coincides with the so-called "geometrical neutral axis" but with load it changes its position—especially in the non-commutating machines. (The machine used is a non-commutating pole motor.) When the brushes are in the magnetic neutral position, the resultant air-gap flux enclosed by them is given by

$$\phi_p = \int_0^\pi B_\theta d\theta \tag{12}$$

where B_θ is the ordinate of the flux distribution curve at an angle θ from the origin as shown in Fig. 5. If now the axis of the brushes is shifted

from its original position α, the effective flux enclosed by them will be

$$\phi_{p\alpha} = \int_{\alpha}^{\tau+\alpha} B_{\phi} d\theta \tag{13}$$

which must always be less than ϕ_p. As α goes on progressively increasing, the integral vanishes at a critical angle of displacement α_C. Physically this must mean that the torque produced in the armature by the north pole flux is cancelled by the torque produced by the south pole flux. Under such conditions, the motor cannot be expected to keep rotating or oscillating and hence it has to come to rest. However, even before the integral vanishes, that is, even before the brush shift angle reaches α_C, the machine breaks away from oscillations reaching very high speeds. From the expression for μ, we have,

$$\mu = \frac{R - \alpha}{nL} \qquad \text{and} \qquad n^2 = \frac{K}{L} \cdot \phi_{p\alpha}{}^2$$

and $\mu^2 = \dfrac{(R-\alpha)^2}{K\phi_{p\alpha}{}^2 L} = \text{constant}/\phi_{p\alpha}{}^2$ provided L is constant. L was observed to be substantially constant for all values of armature current up to the rated current of the oscillator. Thus

$$\mu^2 \phi_{p\alpha}{}^2 = \text{const.}$$

or

$$\mu \phi_{p\alpha} = \text{const.}$$

or

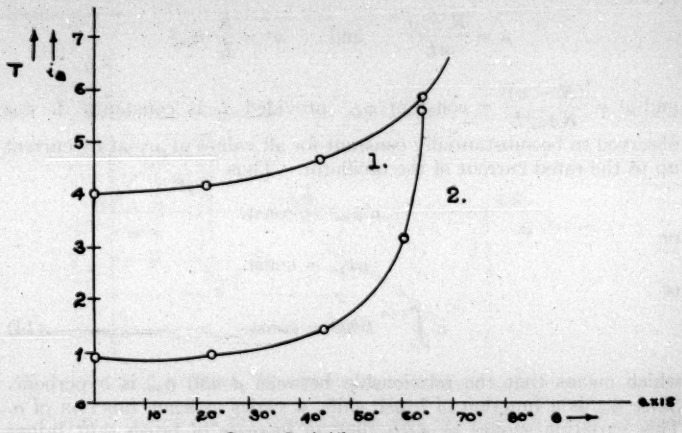
$$\mu \int_{\alpha}^{\tau+\alpha} B\theta d\theta = \text{const.} \tag{14}$$

which means that the relationship between μ and $\phi_{p\alpha}$ is hyperbolic. Since $\phi_{p\alpha}$ is a function of brush shift, μ varies as some function of α. This variation should be such that an increase of brush shift brings about an increase of periodic time and a change in the wave shape which tends towards relaxation type. This was precisely what was observed during investigations (Fig. 6).

If $\phi_{p\alpha}$ becomes small, μ becomes large and period of oscillation also becomes large. There is a critical value for $\phi_{p\alpha}$ at which the periodic time becomes infinite and the machine rotates at a constant speed. Physically any weakening of the total field must result in reduced developed-torque of the oscillator. Consequently the rotor has to take large current and make larger number of revolutions now to develop the necessary back emf. to neutralize the generator voltage and to reverse the direction of rotation. This process necessarily takes longer duration of time. As the field decreases progressively this time tends to-

wards infinity which means breaking away from oscillations. Saturation of the magnetic path accentuates the situation further. During a set of investigations the oscillator broke off from oscillations at a brush shift of about 65° elec.

At this point, it is perhaps interesting to observe that when the brushes were left in their original positions and the field was gradually weakened, a very similar phenomenon was noticed. The motor broke off from oscillations at a very low exciting current of 0.02 amp. But just before doing so, the speed of the motor reached a maximum value of 6500 rpm. while its normal rated speed is only 1720 rpm. After breaking off from oscillations, the machine rotated at about 3500 rpm. The largest periodic time measured was 23 sec. and the largest amplitude of current 18 amp.



Curve 1—Brush shift–armature current.
Curve 2—Brush shift–periodic time.

FIG. 6.

6. EFFECT OF DIVERTING SERIES FIELD CURRENT

A diverter was introduced into the field circuit to measure the changes in periodic time and armature current when the current in the diverter itself was varied by varying the diverter resistance. Figures 7 and 8 show the relationship between (1) diverter current and periodic time and (2) armature current and periodic time. It is of interest to note from Fig. 7 that the periodic time continuously increases as the diverter resistance is gradually reduced while the diverter current itself increases first and then reduces. The curves appear beak-like. Curves (*a*), (*b*), (*c*) and (*d*) plotted for excitation currents of 0.1, 0.14, 0.16 and

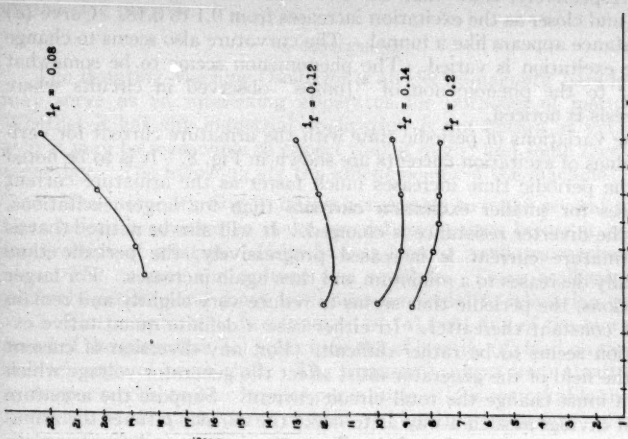


FIG. 8. Curves showing relationship between periodic time and armature current.

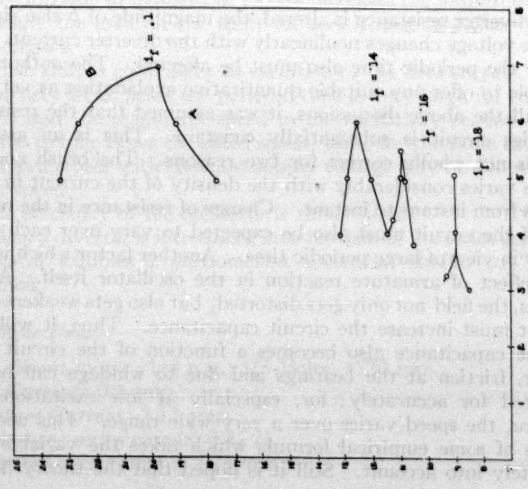


FIG. 7. Curves showing the relation between diverter current and periodic time.

0.18, respectively, show that the two limbs forming the beaks come closer and closer as the excitation increases from 0.1 to 0.18. Curve (*d*) for instance appears like a funnel. The curvature also seems to change as the excitation is varied. The phenomenon seems to be somewhat similar to the phenomenon of "Jumps" observed in circuits where hysteresis is noticed.

The variations of periodic time with the armature current for various values of excitation currents are shown in Fig. 8. It is to be noted that the periodic time increases much faster as the armature current increases for smaller excitation currents than for larger excitations, when the diverter resistance is changed. It will also be noticed that as the armature current is increased progressively, the periodic time gradually decreases to a minimum and then again increases. For larger excitations, the periodic time seems to reduce very slightly and remain almost constant thereafter. In either case a definite quantitative explanation seems to be rather difficult. For, any diversion of current from the field of the generator must affect the generator voltage which in turn must change the total circuit current. Suppose the armature current divides in such a way as to make the current p times the armature current, at any setting of the diverter resistance (p being less than unity) the generated voltage will be given by

$$eg = \alpha(pi) + \beta/3(pi)^3 - \frac{r}{5}(pi)^5.$$

As the diverter resistance is altered, the magnitude of p also changes so that the voltage changes nonlinearly with the diverter current. Consequently the periodic time also must be affected. The author has not been able to offer any suitable quantitative explanation as yet.

In all the above discussions, it was assumed that the resistance of the series circuit is substantially constant. This is an assumption which is not wholly correct for two reasons. The brush contact resistance varies considerably with the density of the current in it which changes from instant to instant. Change of resistance in the remaining parts of the circuit must also be expected to vary over each cycle especially in view of large periodic time. Another factor which is ignored is the effect of armature reaction in the oscillator itself. At higher currents, the field not only gets distorted, but also gets weakened. This in effect must increase the circuit capacitance. Thus, it will be seen that the capacitance also becomes a function of the circuit current. Further, friction at the bearings and due to windage can hardly be accounted for accurately; for, especially at low excitations of the oscillator, the speed varies over a very wide range. This necessitates the use of some empirical formula which takes the variable friction adequately into account. Still it is hoped that the theory developed

here and earlier will serve to give sufficiently accurate results for engineering purpose.

7. CONCLUSIONS

The Rotating Machine Oscillator is an electromagnetic paradox and may serve as an interesting apparatus for purposes of instruction. Whether it has any industrial application is not known as yet; however it may be reasonable to expect that it might find some special use some day. The following are the salient points of the machine:

1. The motor starting from rest accelerates to make several revolutions before it reverses to make several revolutions before it reverses its direction of rotation. If its M.I. is large it oscillates mechanically also. The current and voltage also oscillate. These oscillations are perfectly periodic and range from near sinusoidal to relaxation type.

2. The periodic time is a function of (*a*) field flux of the motor, (*b*) speed of the generator, (*c*) polar moment of inertia, (*d*) load on the shaft and (*e*) angle of brush shift. The frequency ranges from about 2 to 3 cycles per second to 2 to 3 cycles per minute. It is remarkable that this frequency is about the same as that of "hunting frequency" of a synchronous machine.

3. The oscillator initially "crawls" for values of series circuit resistance greater than the critical resistance of the supply generator. Any attempt to arrest the motion of the shaft results in a current surge; but the motor does not oscillate.

4. Any increase of moment of inertia increases the amplitude of the current while the load on the shaft reduces it, but in either case the period increases.

5. Brush shift is equivalent to variation of the field current.

6. For very weak fields the periodic time is very large (of the order of 20 sec.) and the maximum speed in the oscillatory regime may be about three to four times the rated speed, while for the same excitation the speed as a motor with constant supply voltage may be about double the rated speed.

7. The efficiency of the oscillator is rather poor.

8. When a diverter is introduced into the field of the series generator, the oscillator exhibits a phenomenon somewhat similar to "Jump" phenomenon.

The Machine Data

D. C. Shunt motor: 1/4 h.p.; 115 Volts; 1720 rpm.
Field Current: 0.2 amp.
Armature Current: 2.12 amp.
Inductance: series circuit: 0.21 *H*.
Dynamic Capacitance = 4.74 m.F.

Motor—Generator Set

D. C. Shunt motor: 2 h.p.; 220 V; 1050 rpm.
Series Generator: 1 KW; 220 V; 1050 rpm.

REFERENCES

(1) C. V. GOVINDA RAO AND V. V. SARWATE, "Analysis of Rotating Machine Oscillator," *J. Inst. Engineers (India)*, Part 2, Vol. 37, p. 929 (1957).
(2) C. V. GOVINDA RAO, "A Solution of Non-linear Differential Equation of the Rotating Machine Oscillator," JOUR. FRANKLIN INST., Vol. 265, p. 29 (1958).
(3) P. LE-CORBEILLER, "The Non-linear Theory of Maintenance of Oscillations," *J. I. E. E.*, Vol. 79, p. 361 (1936).
(4) — THALER and — STEIN, "Parameter Evaluation for d-c. Servomotors," *Trans. A. I. E. E.*, Vol. 75, p. 410 (1956).

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STUMPFF, KARL. Planet Earth. 1959.

BIBLIOGRAPHY

BEATTY, W. B. AND LANGAN, LEE. Sources of Information on Geology and Mining in the Western States. 1957.
NORTH DAKOTA. RESEARCH FOUNDATION. Bibliography of the Geology and Natural Resources of North Dakota, 1814–1944. 1959.
U. S. DEPARTMENT OF AGRICULTURE. List of Serials Currently Received in the Library of the United States Department of Agriculture as of July 1, 1957. 1958.

BIOGRAPHY

CURTIS, HARVEY LINCOLN. Recollections of a Scientist; an Autobiography. 1958.

BIOLOGICAL CHEMISTRY

SCHACHMAN, HOWARD K. Ultracentrifugation in Biochemistry. 1959.

CHEMISTRY AND CHEMICAL TECHNOLOGY

Advances in Inorganic Chemistry and Radiochemistry. Vol. 1. 1959.
Analytical Chemistry. Vol. 1., 1959.
COLLOQUE DE BIOLOGIE DE SACLAY. The Method of Isotopic Tracers Applied to the Study of Active Ion Transport. 1959.
FLASCHKA, H. A. EDTA Titrations; an Introduction to Theory and Practice. 1959.
HENLEY, ERNEST J. AND BIEBER, HERMAN. Chemical Engineering Calculations. 1959.
HERSHENSON, HERBERT M. Infrared Absorption Spectra. 1959.
INSTITUTE OF PETROLEUM. HYDROCARBON RESEARCH GROUP. Proceedings of the ..... Conference on Molecular Spectroscopy. 1959.
LEVA, MAX. Fluidization. 1959.
LIFSHITZ, EVGENII MIKHAILOVICH AND ANDRONIKASHVILI, E. L. A Supplement to "Helium." 1959.
MILLER, JAMES; GERHAUSER, JOHN M. AND MATSEN, F. A. Quantum Chemistry Integrals and Tables. 1958.
PECSOK, ROBERT L., ED. Principles and Practice of Gas Chromatography. 1959.
SCHWITZER, M. K. Continuous Processing of Fats. 1959.
TROTMAN-DICKENSON, A. F. Free Radicals; an Introduction. 1959.

COMPUTERS

RADIO CORPORATION OF AMERICA. RCA SERVICE COMPANY. GOVERNMENT SERVICE DEPARTMENT. The Language and Symbology of Digital Computer Systems. 1959.
WARFIELD, JOHN N. Introduction to Electronic Analog Computers. 1959.

CRYSTALLOGRAPHY

SARATOVKIN, D. D. Dendritic Crystallization. Ed. 2. 1959.

ELECTRICITY AND ELECTRICAL ENGINEERING

GILLE, J. C. Feedback Control Systems. 1959.
HAUS, HERMANN A. AND ADLER, RICHARD B. Circuit Theory of Linear Noisy Networks. 1959.
INTERNATIONAL UNION OF PURE AND APPLIED CHEMISTRY; SECTION OF ORGANIC CHEMISTRY. Theoretical Organic Chemistry. 1958.
KLEEN, W. J. Electronics of Microwave Tubes. 1958.
KUH, ERNEST AND PEDERSON, DONALD O. Principles of Circuit Synthesis. 1959.
SAFFORD, EDWARD L., JR. Model Radio-Control. 1959.
SMULLIN, LOUIS D. AND HAUS, HERMANN A., ED. Noise in Electron Devices. 1959.

ENGINEERING

BENJAMIN, JACK R. Statically Indeterminate Structures. 1959.
Progress in Non-Destructive Testing. Vol. 1. 1958.
WILLIAMS, CLIFFORD DAVID. Analysis of Statically Indeterminate Structures. Ed. 3. 1955.

FOOD

SOCIETY OF CHEMICAL INDUSTRY. Recent Studies in Yeast and Their Significance in Industry. 1956.

GENERAL

EMME, EUGENE MORLOCK, ED. The Impact of Air Power. 1959.
NYBAKKEN, OSCAR E. Greek and Latin in Scientific Terminology. 1959.

HYDRAULICS

ELEVATORSKI, EDWARD A. Hydraulic Energy Dissipators. 1959.
LANGBEIN, WALTER B. AND HOYT, WILLIAM G. Water Facts for the Nation's Future. 1959.
LELIAVSKY, SERGE. An Introduction to Fluvial Hydraulics. 1955.
LINSLEY, RAY K. Hydrology for Engineers. 1958.
TODD, DAVID KEITH. Ground Water Hydrology. 1959.

INDUSTRIAL MANAGEMENT

ABRUZZI, ADAM. Work, Workers and Work Measurement. 1956.

LANGUAGE

NEVILLE, H. H. AND YUILL, W. E. Translation from German for Chemists. 1959.
TURKEVICH, J. AND TURKEVICH, LUDMILLA B. Russian for the Scientist. 1959.

MANUFACTURE

BOLAM, FRANCIS, ED. Fundamentals of Papermaking Fibres. 1958.

MATHEMATICS

BODEWIG, E. Matrix Calculus. Ed. 2. 1959.
CHENG, CHUN. Analysis of Linear Systems. 1959.
KEMENY, JOHN G. Finite Mathematical Structures. 1959.
LANG, SERGE. Introduction to Algebraic Geometry. 1958.
LOMONT, J. S. Applications of Finite Groups. 1959.

V. 19

SANSONE, G.; DIAMOND, AINSLEY N. AND HILLE, EINAR. Orthogonal Functions. Rev. Eng. ed. 1959.
THIES, MATTHIAS A. Industrial Mathematics. 1959.
WAGNER, THOMAS CHARLES GORDON. Analytical Transients. 1959.

MECHANICAL ENGINEERING

RADZIMOVSKY, EUGENE IVAN. Lubrication of Bearings. 1959.

METALLURGY

BIRCHENALL, C. ERNEST. Physical Metallurgy. 1959.
HEHEMANN, R. F. AND AULT, G. MERVIN, ED. High Temperature Materials. 1959.
INSTITUTE OF METALS. Vacancies and Other Point Defects in Metals and Alloys. 1958.

METEOROLOGY

RAWER, KARL. The Ionosphere. 1952.

MINERALOGY

KERR, PAUL FRANCIS. Optical Mineralogy. Ed. 3. 1959.

NUCLEAR ENGINEERING

BRETSCHER, E. AND HUGHES, D. V. Soviet Reviews of Nuclear Science on the 40th Anniversary of the October Revolution. 1959.
LANDAU, LEV DAVIDOVICH AND SMORODINSKY, YA. Lectures on Nuclear Theory. 1959.
SCHMIDT, KARL RUDOLPH. Nutzenergie aus Atomkernen. Vol. 1. 1959.

OPTICS

CHRÉTIEN, HENRI. Calcul des Combinaisons Optiques. Ed. 4. 1958.
MOSS, T. S. Optical Properties of Semi-Conductors. 1959.

PHYSICS

BRANDEIS UNIVERSITY. SUMMER SCHOOL. Institute in Theoretical Physics. 1957.
ESTERMANN, IMMANUEL. Classical Methods. 1959.
HEAT TRANSFER AND FLUID MECHANICS INSTITUTE, 1959. Preprints of Papers. 1959.
OUZIAUX, R. AND PERRIER, J. Mécanique Appliquée. Vol. 1. 1958.
WHITE, GUY KENDALL. Experimental Techniques in Low-Temperature Physics. 1959.

ROCKETS AND MISSILES

Dictionary of Guided Missiles and Space Flight. 1959.
ROYAL AERONAUTICAL SOCIETY. High Altitude and Satellite Rockets. 1959.

SANITARY ENGINEERING

SALVATO, JOSEPH A. Environmental Sanitation. 1958.

SCIENCE

PLANCK, MAX KARL ERNST LUDWIG. The New Science. 1959.

STATISTICS

BOWKER, ALBERT H. AND LIEBERMAN, GERALD J. Engineering Statistics. 1959.
CHERNOFF, HERMAN AND MOSES, LINCOLN E. Elementary Decision Theory. 1959.
CROXTON, FREDERICK E. Elementary Statistics. 1959.

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E. V. Stewart
Elmer Stilwell
J. E. Sunkes
Charles E. Voorhees
Raymond Wynkoop
Carl K. Zimmermann

EDUCATOR

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Morris Blackburn
J. Frederick Hazel
William H. Henneman
J. Lee McConnell
J. Stanley Morehouse
Helen W. Nutting
Charles E. Sohl
James J. Stekert
Norma J. Wright

NON-RESIDENT

James Bailey
Emanuel Rosenberg

August 1–31, 1959

ANNUAL

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Roger F. Burd
Lewis J. Burger
James E. Clark
Robert I. Cummin
John R. Curtis
Frans G. Dahlin
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Stanley G. Ford
Burton J. Gray
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James C. Hutchison
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Leonard Muldawer
Thomas F. Nealon, Jr.
Vincent W. Uhl

NON-RESIDENT

George L. Morrison
Joshua L. Baily, Jr.

NECROLOGY

Donald Alexander '45
Rodman Barker '44
Thomas R. Bennett '54
John B. Cramp '58
Mills A. Eure '55
Horace C. Jenkins '50
Jack Lutz '55
William Perelstrus '43
Mrs. C. Fred Rau '56
J. Allison Scott '56
Leonard Smidth '59
Charles S. Swope '48
John G. Tappert '58
Russell Harrison Varian '53
Sidney R. Zall '56

BOOK REVIEWS

THE MAGNETO-IONIC THEORY AND ITS APPLICATIONS TO THE IONOSPHERE, by J. A. Ratcliffe. 206 pages, diagrams, $8\frac{3}{4} \times 5\frac{3}{4}$ in. New York, Cambridge University Press, 1959. Price, $7.50.

Since, in 1902, Kennelly and Heaviside independently suggested that radiowaves might be propagated from England to America by reflection from an ionized layer in the upper atmosphere, theoretical contributions by many authors have supplemented the experimental progress on the propagation of the electro-magnetic waves in the ionized gas. In 1925, Appleton and Barnett in England, and Nichols and Schelleng in America pointed out independently and almost simultaneously the importance of the influence of the earth's mangetic field upon the propagation of the radio waves in the ionosphere. Here the name "magneto-ionic" was applied to the theory which was almost thoroughly established by Appleton in 1932.

This book presents, in a most precise manner, a detailed account of this theory. Not only are the fundamental equations derived, but the physical reasons for the behavior of the waves are discussed, with special emphasis upon ionospheric effects. Furthermore, about one-quarter of this book contains material which has not previously been published, and the bibliography lists over 130 references dealing primarily with the magneto-ionic theory and its applications.

Part I deals with the derivation of Appleton's fundamental equations which express the complex refractive indices and the wave-polarizations for waves travelling in a magneto-ionic medium. Two different approaches are used, a macroscopic one in which the properties of the medium are averaged out, and a microscopic one in which attention is concentrated on the motions of the individual elections. These two approaches facilitate the interpretation of the equations.

Part II consists of the graphical representations of the refractive index and absorption index as a function of election density which summarize the results given by the equations.

Part III is concerned with the application of the theory to all of the phenomena associated with the terrestrial ionosphere known by the author.

This monograph is worthy not only as a guide book for persons who are interested in this field, but also as a desk-side book for professionals.

KASLOW NAGASHIMA
Bartol Research Foundation

CONTINUOUS ANALYSIS OF CHEMICAL PROCESS SYSTEMS, by Sidney Siggia. 381 pages, illustrations, 6×9 in. New York, John Wiley & Sons, Inc., 1959. Price, $8.50.

Modern chemical plants require accurate, rapid monitoring of various process streams. The past decade has seen major developments and utilization of instrumentation for this purpose. The object of this book is to present description and specifications for various major instruments with examples of applications.

There are 25 chapters. The first two serve as an introduction and discuss such topics as manpower, economic and timing considerations in setting up analytical schemes. The last chapter discusses instruments in the research and development stage such as the "Time-of-Flight" mass spectrometer, nuclear magnetic resonance, polarimeter, etc. These newer instruments have not been taken out of the laboratory for plant use.

The remaining chapters are divided into physical categories such as viscosity measurement, infrared absorption, ultraviolet absorption, visible absorption, coulometry, vapor chromatography, gas analysis using sonic velocity, etc. Chapter 20, Oxygen, offers an example of the organization of the book. A brief introductory section mentions the importance of the measurement and describes some general analytical approaches. The body of the chapter is divided into four sections: paramagnetic approach, catalytic reaction with hydrogen, effect on a galvanic cell, and chemical oxidative properties. Under each section specific instruments are described, for example under paramagnetic

V.2
195

approach: the Beckman Analyzers F3 and G2, the Hartmann and Braun (Magnos), the Hays (Magno-Therm), the Leeds and Northrup, and the Siemens and Halske (Oxymat).

Actually the book is a summary of manufacturers literature. It would be useful to those requiring or selecting instruments for specific continuous measurements who wish to know what is available and by whom. There is little in the way of explanation of the optical, physical or chemical terms and equipment components used in these devices. It would seem that the inclusion of an explanatory glossary of terms and expansion of the description of the principles of operation would increase the value and acceptance. A volume of this type will require frequent revision since instrumental advances are quite rapid. Design and process engineers will find this edition quite useful. R. A. BAKER

The Franklin Institute Laboratories

ELEMENTS OF MATERIALS SCIENCE, by Lawrence H. Van Vlack. 528 pages, illustrations, 6 × 9 in. Reading (Mass.), Addison-Wesley Publishing Co., Inc., 1959. Price, $8.50.

In general, the student obtains his knowledge of engineering materials from courses devoted to specific classes of materials, for example, metals, polymers, ceramics, etc. A metallurgy student faced with the task of studying the multitude of metals and alloys necessary to his trade would not be apt to take a course in the chemistry department on polymers. Very few engineers have had formal training in more than one or two classes of engineering materials. However, most of them have had to extend their knowledge to a wider range of materials through self-study and experience.

It has been recognized during the past few years that many of the underlying principles involving the understanding and use of engineering materials are common to one another. A few courses are now offered in various universities. This text results from such a course. The book is particularly important because it is one of the very few that treats the materials subject from a broad and scientific viewpoint. It attempts to ground the student in scientific principles and minimizes detailed discourses on specific materials.

The author introduces the subject by acquainting the reader with the various engineering terms and measurements. He then devotes nine chapters to the internal structure of materials and the relation of structure to properties. The latter part of the book is devoted to materials under service conditions of stress, temperature, corrosion, electromagnetic fields, and radiation. The section devoted to the behavior of materials in electromagnetic fields is especially good since it touches upon a subject usually neglected in the early training of an engineer.

In the study of crystal structure it is necessary to illustrate the geometrical relation of atoms within the crystal to each other. The present text is profusely illustrated with outstanding diagrams of this relation. The author also makes good use of structure diagrams to illustrate such phenomena as deformation.

Each chapter has references and problems at the end. Answers are provided to half of the problems and examples are worked out in the text. There are ten appendices occupying seventy pages. These cover such things as tables of elements, constants, conversion charts, definition of terms and selected properties. The beginning student is not apt to make much use of some of the appendices because of their complexity. For instance, an appendix on selected phase diagrams gives thirty-five of them, most of which are too difficult for the beginner to comprehend.

Although the book is designed as an elementary text aimed at freshman and sophomore engineering students, there is much to be learned from it by the metallurgist who has not studied ceramics, organic materials, etc. and conversely by the ceramicist or organic chemist who has not studied metallurgy. It is well organized, well written, and extremely well illustrated. R. L. SMITH

Franklin Institute Laboratories

ROCKET ENCYCLOPEDIA ILLUSTRATED, edited by John W. Herrick and Eric Burgess. 607 pages, diagrams, illustrations, $7\frac{1}{4} \times 10\frac{1}{4}$ in. Los Angeles, Calif. Aero Publishers, Inc., 1959. Price, $12.50.

Within a period of two decades, rockets have exploded from laboratory curiosities manipulated by "mad" scientists to instruments for delivering incredibly lethal payloads thousands of miles from the launch point and

for the conquest of space. From a handful of dedicated individuals there has been such an expansion that hundreds of thousands are engaged in either design, construction or development of these devices. Thus, there has arisen a critical need for a reference book which can answer questions posed by the serious worker or newcomer to the field. As Dr. Theodore von Karman indicates in his foreword: "we realize it is progressively more difficult to keep abreast of all phases of rocketry. The 'solid propellant' man gets so engrossed in the myriads of problems involved in his specialty that he cannot always keep up with the 'liquid propellant' man's problems, and vice versa."

Dr. von Karman stresses the need for an encyclopedia presentation of the material available on specific research and design problems.

Rocket Encyclopedia Illustrated has been written to fill this gap and with it the compilers, John W. Herrick and Eric Burgess, the two ranking authorities have undertaken a most difficult task. They have carefully checked the available data and have selected only those entries necessary for an understanding of rocketry. The result is this new, fascinating and completely essential compendium.

Thousands of entries are alphabetically arranged in the book. The subject leads off with the name of the entry which is followed by the definition of the term. This is followed by the exploration.

A significant feature of the new explanation section is the introduction of some terms in italics. Where the editors believe further clarification is necessary, additional terms are italicized indicating entries.

A further feature which sets this encyclopedia apart from the run of the mill type is the "visual keying" used in the illustrations. Each part of the illustration identified by a call out is also indicated in bold face type where it appears in the accompanying text.

An appendix is included which contains photo credits, a bibliography and references, abbreviations and symbols used in rocketry.

The editors have compiled a much needed and magnificent contribution to rocketry and are to be congratulated on their final product. The nature of this book is such that no school, laboratory or technical station can afford to be without one. It should enjoy a wide distribution.

I. M. LEVITT
Franklin Institute Planetarium

FLUIDIZATION, by Max Leva. 327 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $11.50.

Fluidization is one of the newer unit operations which became important during World War II when it was used by the oil industry to upgrade petroleum stocks to high octane gasolines. Developments have been rapid since that time and fluidized beds have been used by many other process industries.

Until the publication of *Fluidization* only one important book on the subject existed. It was based on a collection of papers and consequently did not adequately cover all aspects of the science. Mr. Leva has previously established a reputation for basic investigations in the mass transfer operations. This book contains the same thoroughness typified by his publications in solvent extraction. There is a very unusual feature in this text. Every chapter is organized so that the sequence of developments and the research approach are explained. The author not only summarizes the important conclusions but indicates those facets requiring additional exploration. Many industrial laboratory studies and thesis subjects are contained in Mr. Leva's summations.

Of the ten chapters, the principle ones describe the fluidized state, fixed bed and the onset of fluidization, the expanded bed, elutriation, dilute phase and moving solids. Related chapters concern the spouted bed, mass and heat transfer, and solids and fluids mixing.

Graphical presentation of analytical data is liberally used to summarize the results. Reference tables, dimensional analyses correlations, and schematic diagrams are abundant. A bibliography of the important literature is appended to each chapter. Some sample problems and their solutions are presented. This will facilitate the use of the book as a text. Additional problems for classroom or student use are offered.

The author avoided a strictly "application viewpoint" in preparing *Fluidization*. Fundamentals have been stressed. This is wise since future development may be expected and subsequent editions will be easier to update.

Chemical engineers and students of the fluidization unit operations will find this to be a valuable addition to the technical literature.

R. A. BAKER
The Franklin Institute Laboratories

V.2
195

STRUCTURAL DESIGN FOR DYNAMIC LOADS, by Charles H. Norris, Robert J. Hansen, Myle J. Holley, Jr., John M. Biggs, Saul Namyet and John K. Minami. 453 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $12.50.

In the past, dynamic loads on civil engineering structures have been small in comparison with static loads, so that it has rarely been necessary to evaluate them precisely. This situation has changed in recent years, as live loads have tended to become more severe. Among these dynamic loads are those caused by heavier machinery running at higher speeds, and greater wind loads on higher towers and longer bridges. Moreover, the development of more powerful explosives, particularly of nuclear weapons, and the experience of widespread damage from earthquakes, have stimulated a trend towards application of dynamic principles in structural design. The present volume is intended as a survey of this growing field.

The first three parts, comprising about half of the book, review the analytical tools which are applied in the second half. Part 1 deals with the behavior of steel and reinforced concrete elements such as beams, columns and slabs under dynamic loading. Part 2 treats the calculation of the response of structural systems to a variety of dynamic load conditions. In its completeness and range of subjects covered, it is almost a book in itself. Included are such topics as the dynamic response of one-degree-of-freedom systems, and the general theory and detailed analysis of concentrated and distributed mass systems. A separate chapter on dynamic response theory discusses the effects of impulsive and impact loads, non-linear and elasto-plastic stiffness, and damping. It stimulates further research by pointing out the deficiencies of fundamental theories of impact loading which will be needed for the design of protective construction against missile fragments. The final chapter of Part 2, nearly 50 pages long, clearly illustrates simplified methods of analysis and design for dynamic loads by means of many fully worked-out examples.

Part 3 deals with computational methods and their application to response calculations. It includes a review of numerical integration methods and gives a brief review of analog and digital computers, as used in this type of analysis.

The second half of the book, Part 4, is devoted to applications of the theory to specific cases of dynamic loading. The principal part, chapters 10 to 15, which deals with blast-resistant design, is a condensation of a design manual prepared by the Massachusetts Institute of Technology for the Office of Chief of Engineers, U. S. Army. It takes into account data on weapons effects, including effects of nuclear weapons, such as blast, thermal and atomic radiation. These data were collected by the U. S. Strategic Bombing Survey in Japan, and by the Federal Civil Defense Administration from tests in Navada.

Chapter 14 presents two examples of blast resistant design, a one story steel frame building, and a one-story reinforced concrete shear wall building. Chapter 15 briefly discusses pertinent laboratory and theoretical developments, among them the MIT Shock Tube, the MIT rapid loading machine, the University of Illinois pulse loading machine, and other special devices still under construction.

In the next three chapters, Professor John K. Minami of Waseda University, Tokyo, discusses dynamic loads due to earthquakes. He illustrates their effect by the collapse of the Daiwa Department Store in 1948. The Japanese building code is reviewed, and modifications are suggested. Earthquake-resistant design methods, both dynamic and static, are summarized and design data on an eight-story reinforced concrete building are used to illustrate and compare the various methods described.

The final chapters of the book review two other important sources of dynamic loading: those due to vibration of girders under moving traffic loads, and the effects of wind loads due to aerodynamic interaction between the moving structure and the air stream.

The book may be considered an excellent introduction to rational design for dynamic loads. One may hope that the authors will extend their work, or stimulate others to do so, for the benefit of other areas of structural design, such as, e.g., naval architecture, where more refined methods of analysis would be useful.

K. L. CAPPEL
The Franklin Institute Laboratories

GROUND WATER HYDROLOGY, by David K. Todd. 336 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1959. Price, $10.75.

Dr. Todd observes in his preface that few men specialize in ground water hydrology, yet, because water is a major natural resource it is important to a variety of fields. *Ground Water Hydrology* presents a lucid account of the physical fundamentals, methods of analyzing and solving related problems, and historical background of the various phases of this subject. It has one of the clearest, easiest-to-read styles of any technical book this reviewer has seen in years. Students and experienced workers in the field will find the descriptions of well development, recharge of ground water, and sea water intrusion almost narrative in style.

The book is in fourteen chapters divided to present topics such as ground water occurrence, movement, hydraulics, quality, basin wide development, surface and subsurface investigations, legal aspects and model studies. Mathematical statements for hydrological principles are thorough but without an overburden of unnecessary derivations. Pictorial and graphical explanations of the subject matter are generously utilized. Bibliographies are extensive and are appended to each chapter.

Ground Water Hydrology would make an excellent text for a course of that title although there are no sample or assignment problems. Civil, sanitary, geological and agricultural engineers and others with interest in water resources should find it a complementary treatise.

ROBERT A. BAKER
The Franklin Institute Laboratories

VISTAS IN ASTRONAUTICS, edited by Morton Alperin and H. F. Gregory. 318 pages, Vol. 2, illustrations, $7\frac{1}{4} \times 9\frac{1}{4}$ in. New York, Pergamon Press, 1959. Price, $15.00.

In the spring of 1958 the Second Annual Air Force Office of Scientific Research Astronautics Symposium was held in Denver, Colorado. From all over the country came engineers and scientists to present papers or be auditors in a resumé of the state of the art in space travel.

Events in the field are moving so rapidly that just to survey the literature assumes the proportions of a full time job. Thus any place where a comprehensive survey is to be found becomes a valuable reference source.

Such a reference work is *Vistas in Astronautics*, Vol. 2. While not all facets of the space travel field are covered—this is an impossibility—those which are covered bring the reader up-to-date at least to the spring of 1958.

The book is divided into five parts dealing with: 1. Space Environment and Vacuum Research; 2. Control and Propulsion of Vehicles Outside the Atmosphere; 3. A Panel Discussion on Satellite Capabilities; 4. Departure, Space Navigation and Re-entry Problems; and 5. The Earth's Moon. Even in these sections there is a variety of topics covered which in some instances stretch the title.

In Part I, Richard Roche and Otto Schueller cover simulation of space conditions. Roche has participated in the construction of the Litton Industries device which simulates to an altitude of 90 miles. Schueller presents the need for simulators to meet various space conditions which he defines in a most satisfactory way. Johnson and Singer set down the experiments that could be performed from a lunar vehicle and the scientific uses of a Cislunar vehicle. Singer points out that the height of the lunar bulge is 2200 meters (about 7400 feet) and indicates there is serious doubt concerning this figure which Baldwin uses. This has been redetermined as 3600 feet. Thus, the experiment Singer proposes may resolve this uncertainty.

Part II contains an excellent paper by R. M. Patrick on a magnetohydrodynamic propulsion motor. This motor is shown to have performance characteristics similar to the ion rocket motor. This apparently new concept is important because it indicates that our engineers are still seeking exotic sophisticated propulsion systems.

Part III provides the philosophical facet of the meeting, but it is sound philosophy and must enter into any discussion on the utility and capability of man-in-space. As might be expected, the experts are certainly not unanimous in believing that scientific missions require a human passenger.

Part IV involves propulsion systems of various types. Here is also discussed the re-entry problem for both satellites and ballistic missiles.

To this reviewer the last section is of greatest interest. It consists of four papers on the moon. Dr. Barnes presents a brief section on the scientific and engineering exploration of the moon. Dr. Thomas Gold in his paper presents one side of the controversial "dust on the moon" question. His thesis is that there is a deep layer of dust present on the moon. Immediately following this we find Dr. Fred Whipple's excellent paper on the lunar surface; he believes this deep dust layer is lacking. Finally, Dr. Gerard P. Kuiper presents his exploration of the moon. This last paper is a must for all interested in lunar features and detail. Dr. Kuiper has spent much time and energy researching lunar formations, and in this section his researches paint a picture of the past history of the moon. Some of the finest photographs of lunar detail illustrate Dr. Kuiper's paper.

Vistas in Astronautics, Vol. 2 belongs in the library of those who would keep abreast of developments in this revolutionary field.

I. M. LEVITT
Franklin Institute Planetarium

268
59

LIQUID HELIUM, by K. R. Atkins. 312 pages, diagrams, 5½ × 8½ in. London, Cambridge University Press, 1959. Price, $11.00.

Liquid helium is a fascinating substance. Composed of simple, monatomic molecules, interacting only through very weak van der Waal's forces, here is a material which by all reasonable expectation ought to be the very archetype of a simple fluid composed of hard spheres. On the contrary, it has properties which are so remarkable that after fifty years of investigation fruitful experiments may still be performed with relatively simple equipment. Liquid helium (He^4, the common stable isotope) has a critical point at 5.2°K, and 2.26 atmospheres pressure. From the critical point to the λ-transformation, which, under the saturated vapour, occurs at 2.17°K, the liquid is called helium I. Helium I is not spectacular, although it does have some atypical properties, among which is a positive temperature coefficient of viscosity. Below the λ transformation to the lowest temperatures reached, and presumably to absolute zero, the liquid phase persists. This liquid is called helium II.

Helium II is unique in many ways, though perhaps the most striking single property is its "superfluidity," by which it is able to flow without friction through extremely narrow cracks and pores. This "superfluidity" has been termed a macroscopic manifestation of the quantum behavior which is characteristic of microscopic systems.

In a brief introductory chapter, Prof. Atkins gives in broad outline the more unusual properties of liquid helium. This is followed by a detailed account of the equilibrium properties in chapter two. Chapter three contains an excellent summary of the theories which have been advanced to explain the equilibrium properties. Chapters four through seven are devoted to the transport properties of liquid helium and to the new thermohydrodynamics which must be developed to explain effects such as superfluidity, second sound, and the peculiar viscous effects found in rotating liquid helium II. Chapter eight is devoted to the strange twin of liquid He^4, namely liquid He^3. The stable light isotope of helium is present in well helium to the extent of about 1.4 parts in 10^7 but is now available in reasonable quantities from the radioactive decay of reactor produced tritium. The properties of the liquid He^3 are complementary to those of liquid He^4. There is no λ transformation or "superfluidity" but the liquid state persists apparently to absolute zero. The ninth and last chapter is devoted to solutions of He^3 in liquid He^4 and to the additional insight into the properties of liquid helium which can be obtained from studies of these solutions.

It is a pleasure to review a book on this subject by an author who has himself contributed much to our understanding of the properties of liquid helium, and whose infectuous enthusiasm should arouse a response in even the most casual reader. This book will be an asset on the bookshelf of any physicist, and invaluable to anyone contemplating research at liquid helium temperatures, or on the theory of the liquid state. There is one small criticism, and this is directed toward the publisher, rather than to the author. Why are most of the figures on the overleaf?

F. J. DONAHOE
The Franklin Institute Laboratories

BOOK NOTES

RADIATION HYGIENE HANDBOOK, edited by Hanson Blatz. 926 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $27.50.

This new handbook covers industrial, medical and research uses of radiation and atomic energy, with applications to the fields of health physics, industrial hygiene and sanitary engineering. It is concerned with the safety aspects of radiation sources, waste disposal, air and water pollution by radiation, and physiological effects of radiation.

There are 23 sections, the last 20 of which cover specific radiation topics. The first three comprise general reference data (weather, squares and cubes, log tables, a table of half-lives of radioactive elements, etc.), a glossary of terms, and a section on exposure standards and radiation protection regulations. In addition to the general tables of data, several chapters contain valuable tables; for example, Section 6 on Radioactive Isotopes and Their Properties is a 190-page compilation in tabular form and Section 8 on Radiation Attentuation Data has 62 tables and 25 graphs. Some sections, notably the one on Liquid and Solid Waste Disposal, are well referenced; others do not list any source material. Section 19, Physiological Effects of Radiation, is a 17-page treatment of radiation and its effects on the human body.

The handbook has been planned to serve those responsible for safety in the manufacture, transportation or disposal of radioactive wastes, for the protection of workers, equipment and material, as well as the public.

DIGITAL COMPUTING SYSTEMS, by Samuel B. Williams. 229 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $7.75.

A broad technical introduction to digital computing systems is presented in this text, especially written for those with engineering backgrounds and for others possessing familiarity with electrical circuits and apparatus. Ten fact-filled chapters contain everything from coding and components to computer applications. The author describes the various elements contained in modern computers, explaining the underlying principles of the devices used and illustrating the devices themselves. The circuitry by which the elements are made to perform desired functions is clearly covered, with many examples of typical circuits. Logical design and programming are discussed as well as a brief picture of how digital computers can be used to help solve scientific, business and data handling problems. With more than 150 carefully selected illustrations giving a wide pictorial survey of available equipment and components, the author hopes to give the reader a sound technical grasp of what computing systems contain and how they accomplish their purpose.

PRINCIPLES OF PHYSICAL CHEMISTRY, by William H. Hamill and Russell R. Williams, Jr. 607 pages, diagrams, 6 × 9 in. New York, Prentice-Hall, Inc., 1959. Price, $8.75.

This new volume offers a clear and unified treatment of every phase of physical chemistry. Chapters on classical thermodynamics, chemical equilibrium and basic material on chemical kinetics are presented in a logical and easily comprehensible sequence. Other important features of this text receiving intensive discussion, are phrase equilibria and colligative properties. Exercises at each point are given with answers provided, and clear, well-integrated summaries are included at the end of each chapter. A useful contribution for those wishing an introduction to the field of physical chemistry, as well as an up-to-date reference on it.

RECENT RESEARCH IN MOLECULAR BEAMS, edited by Immanuel Estermann. 190 pages, diagrams, 6 × 9 in. New York, Academic Press, 1959. Price, $6.50.

The ten papers in this volume were prepared as a tribute to Otto Stern, founder of

molecular beam research, on his 70th birthday. The volume is not designed to be a comprehensive coverage of the subject, since the authors' principal objective was to express their appreciation of Stern's contributions. Two of the papers describe early work in the field, one at Hamburg and the other at the University of California; the other eight fall into two categories—review papers and research papers. An example of a review paper is the one from Oak Ridge on "Some Applications of Molecular Beam Techniques to Chemistry," and an example of the research group, also from Oak Ridge, is "A Stern-Gerlach Experiment on Polarized Neutrons," which is reprinted from *Physical Review*. All the papers serve to illustrate the various fields in which Stern's work has been influential.

AUTOMATION, CYBERNETICS AND SOCIETY, by F. H. George. 283 pages, plates, 5½ × 8½ in. New York, Philosophical Library, 1959. Price, $12.00.

Aimed particularly at the layman, this book describes the development of automation and cybernetics and deals with the question of what can be expected of them in the future. The first chapter, "Science and Common Sense," is an interesting history of the resistance by man to changes in his way of living; against this background of "inertia" the author poses the question of whether the effort to change society to an automated one is worth while. The main body of the book deals with definitions and explanations of cybernetics, communication theory, logic and machines, servomechanisms, feedback, computers, etc. The final section considers the problems and social effects liable to be brought about by widespread use of automation. The final chapter, "Future of Civilization," contains some thought-provoking ideas on what will happen to religion, the arts, and society under an automated civilization.

THE DIESEL ENGINE, by L. V. Armstrong and J. B. Hartman. 360 pages, illustrations, 5½ × 8½ in. New York, The Macmillan Co., 1959. Price, $6.50.

While a technical book can hardly be expected to have a plot, this one has a logical framework on which the facts are supported. As a means of pointing up the practical value of the theory, the authors have integrated into the text a basic design project. It is not actually a problem in design, but a step-by-step presentation of the line of reasoning showing how an engine intended for a particular service takes the physical form that it does. What is believed to be the first attempt presented in a book of a mathematical analysis of how a dual fuel and a gas diesel work, is included. At the end of the book problems for each chapter are given.

PUBLICATIONS RECEIVED

FROM EUCLID TO EDDINGTON, by Sir Edmund Whittaker. Unabridged and unaltered republication of the first edition. 212 pages, 5⅛ × 8 in. New York, Dover Publications, 1958. Price, $1.35 (paper).

SPACE & TIME, by Hans Reichenback. English edition, translated by Maria Reichenbach and John Freund, from *Philosophie der Raum-Zeit-Lehre.* 295 pages, diagrams, 5⅛×8 in. New York, Dover Publications, 1958. Price, $2.00 (paper).

CONCEPTUAL THINKING, by Stephen Körner. Unabridged and corrected republication of the first edition. 301 pages, 5⅛ × 8 in. New York, Dover Publications, 1959. Price, $1.75 (paper).

PHILOSOPHY AND THE PHYSICISTS, by L. Susan Stebbing. Unabridged and unaltered republication of the first edition. 295 pages, 5⅛ × 8 in. New York, Dover Publications, 1958. Price, $1.65 (paper).

EXPERIENCE AND NATURE, by John Dewey. Unabridged and unaltered republication of the second edition. 443 pages, 5⅛ × 8 in. New York, Dover Publications, 1959. Price, $1.85 (paper).

CHEMICAL ENGINEERING ECONOMICS, by Chaplin Tyler and C. H. Winter, Jr. Fourth edition, 192 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $7.00.

MATHEMATICS APPLIED TO ELECTRICAL ENGINEERING, by A. G. Warren. Second edition, revised and enlarged, 464 pages, diagrams, 5½ × 8½ in. New York, Plenum Press, Inc., 1959. Price, $11.50.

BASIC ENGINEERING METALLURGY, by Carl A. Keyser. Second edition, 507 pages, diagrams, 6 × 9 in. New York, Prentice-Hall, Inc., 1959. Price, $8.50.

ELECTROLYTE SOLUTIONS, by R. A. Robinson and R. H. Stokes. Second edition, 559 pages, diagrams, 5½ × 8½ in. New York, Academic Press, Inc., 1959. Price, $11.50.

ON MATHEMATICS AND MATHEMATICIANS, by Robert Edouard Moritz. Unabridged and unaltered republication of the first edition originally entitled "Memorabilia Mathematica or The Philomath's Quotation-Book," 410 pages, 5⅛ × 8 in. New York, Dover Publications, 1959. Price, $1.95 (paper).

HOW PHOTOGRAPHY WORKS, by H. J. Walls. 351 pages, diagrams, 5½ × 8½ in. New York, The Macmillan Company, 1959. Price, $8.50.

CURRENT TOPICS

Pushbutton Phone.—A pushbutton telephone device that may eventually replace the rotary dial as the link between people and the telephone switching machine is being developed at Bell Telephone Laboratories.

Pushbutton calling has been made feasible by modern electronic advances, such as the transistor, applied in an extensive development program. Engineers have designed complex equipment to adapt central office switching to pushbutton signals. At

the same time, the appearance and operation of the pushbutton telephone have, in so far as possible, been tailored to suit people's preferences and capabilities, as revealed in laboratory tests.

To place a call on a pushbutton phone, the user simply presses buttons instead of turning a fingerwheel. As each button is pressed, two simultaneous musical tones are generated and transmitted to the central office. The tones, which can be heard on the receiver, are similar to those now heard in some localities when long-distance calls are made.

This is, in effect, a new language, different from the language the dial uses in directing calls through the switching system to their destinations.

The present switching system—which, nationwide, comprises the world's largest computer—does not yet understand this new language. But because of the greater speed and convenience of the pushbutton system, effort has been directed toward making the system bi-lingual. An array of mechanical-electronic equipment, appropriately called a "translator," has been developed for the purpose of adapting existing central offices.

Simultaneously, research has been undertaken to discover which kind of pushbutton set people would like most and could use most rapidly and accurately. This is known as designing for the human factor—suiting the machine to the human.

Hundreds of persons—including delicately manicured women, stubby-fingered he-man types, and near-sighted persons—have helped engineers at the Laboratories find out which button arrangements, size and "feel" might be best.

The selected arrangement of ten buttons—one for each digit—consists of three horizontal rows of three each, plus the zero or "operator" button at the bottom.

A pushbutton mechanism has been efficiently designed to fit into the same shell that now houses the standard instrument, simply replacing the rotary dial so that no extra apparatus is needed at the user's station.

In the electrical circuit of the telephone is a transistor oscillator which generates a different coded pair of tones for each button. The invention of the transistor, announced by Bell

Laboratories in 1948, is one of the advances which make the pushbuttons feasible.

The manual operation of signaling with pushbuttons takes only a fraction of the time needed for present dialing. The average time for tapping out a seven-digit number is about five seconds, although some persons can do it in less than two seconds. Rotary dialing, by contrast, takes an average of nine seconds.

The time taken for a pushbutton call to go through after the number has been signaled would depend on the type of switching system used in the locality. Where crossbar switching has been installed the time would be proportionately reduced. Where step-by-step switching systems operate, the number would be stored momentarily in a memory unit in the central office and the call would go through at slightly better than normal dialing speed.

Laboratory tests indicate that most people are enthusiastic about the pushbutton operation. Further information on the potential general acceptance of a pushbutton telephone is being measured in current customer tests with pushbutton sets temporily placed in the offices and homes of 200 persons in Hamden, Conn., and 200 more in Elgin, Ill. Translating equipment has been installed in the central telephone offices of the two cities.

During human-factors tests of pushbutton sets, individuals were seated in office-type surroundings, where they used sets with various pushbutton arrangements and characteristics. Their performances and preferences were recorded.

Unexpectedly, it was found that a "snap" action did not produce any greater accuracy than a smooth button-action did. Because most users were found also to favor buttons that depress with a very light, smooth pressure, engineers are accordingly developing switches that operate easily, yet are sturdy.

The tests revealed that most persons have very definite preferences concerning pushbutton arrangement, but it developed that variations had less effect on performance. This finding helped determine the latitude of engineering design.

For example, in early tests dozens of button arrangements were tried out—groups of buttons lined up in rectangles, in circles, triangles and even a cross. Various number sequences were tried within these arrays, involving in all a host of possibilities.

Several basic arrangements appeared to give approximately equal performance. Of these, three were about equally well liked. They were a rectangular array of two horizontal rows, a circle, and the three-by-four rectangular array. The last was selected as having technical advantages over the others.

Americans of the pushbutton era, it was found, dislike a button that has a "tinny" sound when pushed all the way down, and most prefer a definite stop, slightly cushioned. A desirable button must also be large enough to display numbers and letters well. Yet accuracy is impaired when a button is too large a "target" at which to aim and punch. The best size turns out to be about half an inch square.

The buttons must also be spaced sufficiently so that users will not strike two at once, yet must be grouped closely enough so that all the buttons can be seen at a glance—about a quarter inch apart.

Records kept on each individual participating in the tests showed that prolonged practice did not significantly improve an individual's accu-

racy, but that it did improve his speed. And a person who was naturally rapid and accurate with one set of push-buttons was usually also more proficient than the average in the use of any other set.

Tunnel Diode.—The "tunnel diode," little over a year old, is both better understood and closer to commercial application as a result of an intensive research program, according to Dr. Guy Suits, General Electric vice president and director of research.

The tunnel diode, first reported in 1958 by Japanese scientist Leo Esaki, is first cousin to a transistor, but operates on a different principle and offers advantages that the transistor does not. Before long it should find its way into high-speed computers, television sets, communication equipment, nuclear controls, satellites and space vehicles, Suits predicted.

"Because we were convinced that the tunnel diode could lead to revolutionary changes in the electronics industry, we launched a concentrated research program on the subject," Suits said. "This has resulted in important new scientific understanding of the device, which has led to vastly improved tunnel diodes, as well as a number of significant improvements in the field of applications.

"Further advances in the design and application of tunnel diodes can be expected, and it is probable that transistors and other solid-state devices will also benefit from the new knowledge."

The improved tunnel diodes are still in the experimental stage, and are not yet commercially available. However, to spur progress in circuit design, General Electric's Semiconductor Products Department now has plans to offer limited quantities of experimental samples for such use within the next few months. All of the research was financed by the General Electric Company itself, Dr. Suits stated.

One of the most significant advances in scientific understanding of the device originated with some observations of mysterious "wiggles" in performance curves. These were first noted by Drs. Nick Holonyak, Jr., and Arnold Lesk at GE's Advanced Semiconductor Laboratory, in Syracuse, N. Y. A theory that successfully explained the puzzling effect was subsequently worked out at the General Electric Research Laboratory in Schenectady, N. Y., by Drs. Jerome J. Tiemann, Robert N. Hall, and Henry Ehrenreich.

The tunnel diode takes its name from the physical phenomenon that makes it possible: "quantum-mechanical tunneling." The term is used to describe the manner in which the electrical charges move through the device. Such motion takes place with the speed of light, in contrast to the relatively slow motion of electrical charge carriers in transistors.

The high speeds at which electrical charges travel in the tunnel diode make it possible for the device to operate at extremely high frequencies. Oscillation frequencies higher than 2000 megacycles have already been obtained, matching advanced transistor performance, and frequencies of more than 10,000 megacycles are expected in the near future.

The device's high-speed response also suggests applications in computers. When used as switches, tunnel diodes have functioned in a fraction of a milli-microsecond—from 10 to 100 times as fast as the fastest transistor.

The device also resists the damaging effects of nuclear radiation. Because it is less dependent on the structural

perfection of its crystal than is the transistor, it is much less affected by the damage that radiation can do to such crystal structures. In this respect it outranks transistors by more than 1000 to 1. Semiconductors that have been used by GE scientists for making tunnel diodes include silicon, germanium, gallium arsenide, gallium antimonide and indium antimonide.

The tunnel diode is smaller than a transistor and, because of a simpler structure, ultimately will be a small fraction of its present size. It also is little affected by environmental conditions. Silicon tunnel diodes made by General Electric work at temperatures as high as 650 F; conventional silicon diodes will not operate above 400 F.

As an electrical circuit element, the tunnel diode exhibits a unique combination of electrical properties including what is called a "negative resistance" over part of its operating voltage range. These characteristics allow it to be used in a wide variety of applications, such as an amplifier, a generator of radio-frequency power, and a switching device. The simplicity of this device makes possible the development of "integrated circuits," in which entire circuits for some applications may be formed on a single semiconductor structure. It is superior to vacuum tubes and transistors for applications in low-noise amplifiers and mixers for high frequencies. Many parametric amplifier jobs, for example, could be performed more easily by tunnel diodes.

Cryotron Research.—The first operating circuit of superconductive, deposited-film, electronic devices was announced at the recent meeting of the Solid State Devices Research Conference at Cornell University. This accomplishment by an Arthur D. Little, Inc., team under Albert E. Slade marks the completion of the second stage in the cryotron research program to produce an electronic switching circuit that can reduce drastically computer construction costs.

When the late Dudley Buck, of M.I.T., demonstrated his first successful cryotron on Christmas Day, 1954, it was quickly recognized as one of the first practical applications of superconductivity. Several research teams were soon busy developing the potentialities of this device.

Operating in liquid helium four degrees above absolute zero (4 K. or −450 F.), the device consisted essentially of only two parts: a "gate" of tantalum wire nine thousandths of an inch in diameter, and a "control" of niobium wire three thousandths of an inch thick coiled around the gate. Superconductivity in the gate permitted indefinite current flow until resistance was restored by the magnetic field set up by current in the control. The cryotron thus performed the function of a switch, like a transistor.

Two questions dominated early research on the device: Is it feasible? and In what ways might it be superior to other switches for computers? Reliability could prove to be an outstanding characteristic of the cryotron. Theoretically there is no predictable end life as there is with vacuum tubes and transistors. For computers employing thousands of electronic switches, such potentially high reliability would be a major advantage. Only the study of completed machines could turn up other possible reliability problems such as maintaining the cryogenic environment.

Other questions, too, required answers such as: would the cryotron be cheaper to produce, operate faster,

prove more suitable for miniaturization? The search for answers has lead during the past five years to the exploration of a wide variety of engineering problems, of suitable metals and their alloys, and of the various circuits appropriate to the cryotron.

The wire-wound device was the object of study during the first research phase. After development of a machine to produce large quantities of the cryotron, came a spot-welding process to assemble the tiny devices with wires so fine that the operator required a microscope. By 1957 the ADL group had constructed a catalog memory containing 1800 cryotrons, which demonstrated that it was feasible to construct operating cryotron circuits. But miniaturizing the element to increase its speed had brought difficult production problems.

Research on the wire-wound device led to the conclusion that circuitry constructed in this manner operated too slowly and was too expensive. ADL therefore directed its attention to making cryotrons of thin superconductive films deposited on a glass substrate. Deposited film cryotrons, it was hoped, would open the way to the construction of entire circuits at one time that would prove faster and cheaper.

Vacuum deposition of vaporized metals to produce films 2500 angstoms (.00001 in.) thick offers such substantial advantages over the old method that the research efforts had clearly entered a new phase of development. First it was necessary to develop techniques for depositing superconducting thin films and measuring their characteristics. At the same time Slade's team set about acquiring a better understanding of the physics of thin film superconductors. For example, until they knew the current distribution in a thin film superconductor, the scientists could not predict how current-carrying capacity would vary with the width and thickness of the films.

Among the advantages of vacuum deposited cryotrons are the reduction of inductance for greater speed, the achievement of radical miniaturization, and comparative ease of fabrication. Tests to date indicate that cryotrons may soon be competitive with other electronic switches in speed. Already one thousand times faster than their wire-wound predecessors, the thin film devices can be made of alloys and combined in circuits that hold promise of speeds on the order of 100 millimicroseconds.

The use of masks, or stencils, in the deposition process permits the construction of complete circuits at a time and hence of far greater miniaturization. Constructed with layers of lead, tin, and silicon monoxide (for insulation), a circuit of ten deposited cryotrons requires only a few micrograms of metal. A conservative estimate suggests that 2000 cryotrons could be contained in a cubic inch. Constructing entire circuits from masks opens the possibility of automation. While making the etched masks may take up to a month, they can be used again and again. One of the outstanding advantages of thin-film circuitry will be the ability to reduce drastically computer construction costs.

When Slade and his colleagues demonstrated for the first time that thin film cryotrons are feasible by producing an operating circuit, they took a positive step toward the day when thin film devices will be used in a superconductive computer. Deposition techniques have been learned. Circuits can be made, and a further increase in their speed is imminent. Scientists must now improve their understanding of the characteristics

of the thin film device, concentrating their attention on such problems as its reproducibility and improvement of superconductive-to-resistive transition of the switch. Alloys may hold a key to improved speed after examination of the characteristics of a wide range of possibilities. Circuits that will make better use of the deposited cryotron's characteristics offer another road to improved speed. Design and testing of new ideas are the third part of the current research phase. The present state of the art indicates that the first cryotron computer can be constructed in the next four or five years.

Natural-Gas Air Conditioner.—A unique water chilling machine, powered by natural gas, has gone into operation to air condition the display areas of Milwaukee Gas Light Company's main office building. Replacing the customary electric motor or steam turbine power source, this centrifugal cooling unit driven by a piston engine using natural gas as the fuel is one of the first installations of its type in the nation.

Installed to dramatize gas as an energy source, it operates at a cost which compares favorably with other types of systems. At 6 cents a therm —the Company's rate for gas used for cooling—the Waukesha engine's operating cost is equal to what would be charged if electricity could be purchased at less than 1 cent per kilowatt hour.

The system makes use of a standard Carrier centrifugal compressor, condenser and cooler with interconnecting piping of 150 tons cooling capacity.

Unique Welder for Missiles.—Development of a welding machine to weld hard-to-weld thin gauge materials for support of the U. S. Army missile program has been announced by the U. S. Army Engineer Research and Development Laboratories, Fort Belvoir, Va.

Trailer-mounted and gasoline-engine-driven, it is a shielded inert gas arc welder that differs from the conventional d-c. welder in that it furnishes both a-c. and d-c. welding current for metallic arc or tungsten inert gas welding. This unit also provides d-c. for metal inert gas welding and consumable electrode process welding by attachment of any of the several commercially available wire feeders and consumable wire guns. This includes push or pull and water or air-cooled type guns up to 300 amp.

Other features built into this welder are 1 KW, 115 volt d-c. auxiliary power while welding; 10 KW, 115/230 volt a-c. single-phase, 60-cycle power; built-in high frequency stabilizer that can be used continuously or for start only, as required for the welding method employed; automatic inert gas control with solenoid valve and post-flow timer; a three-position selector switch to provide a-c. welding or the choice of either straight or reverse polarity d-c. welding, and a built-in and concealed 20-gal. capacity coolant system with electric pump to supply cooling liquid for welding as required.

The weld and power generator is comprised of an engine-driven, revolving-field, single-phase alternating current with 115 volt d-c. excitation provided by a directly coupled exciter. The welding power is fed through the necessary controls and a heavy duty semimetallic rectifier to provide d-c. welding current.

The over-all dimensions of the welding unit, exclusive of trailer, are: length, 59¼ in.; width, 25½ in., and height, 41¼ in. It is mounted on a ¼-ton military trailer chassis, also developed by the Laboratories. The

complete unit weighs approximately 2100 lb.

The welder, which has yet to undergo Signal Corps radio suppression tests, was developed jointly by the Laboratories and the Miller Electric Company, Appleton, Wisconsin. It is currently being used by Engineer and Ordnance units supporting the Redstone missile program.

"Ablation" Permits Higher Heat Tolerance in Plastics.—Reinforced plastic test samples, shaped into miniature nose cones, have proved to be highly resistant to heat during ablation studies conducted at the Air Research and Development Command's Wright Air Development Center, Dayton, Ohio.

Used to overcome missile re-entry problems, the ablation or melting technique dissolves the plastic nose cone coating at an even rate. Excessive heat is absorbed and the missile nose retains a pointed shape during the process.

The new plastics are favored for their superior thermal and mechanical properties. In a simulated missile re-entry into the earth's atmosphere, the plastics successfully endured 15,000 F. (half again as hot as the surface of the sun) for 30 sec. This degree of tolerance to high temperatures is possible because of a heat protective mechanism known as ablation, peculiar principally to organic reinforced plastics.

Ablation is one method of using the heat generated by aerodynamic friction. The material subjected to this environment may change from a solid to a liquid or gas; change from a liquid to a gas; undergo chemical decomposition or erode. Which of the above actions takes place depends upon the temperature and velocity of the gases.

Continuous Production of Pressure-Resistant Plastic Piping.—The manufacture of reinforced thermoplastic tubing by a new high-speed and continuous process may easily revolutionize and greatly expand the potentials of the plastic piping industry. The process, developed in France by Dr. René Imbert and Mr. Pierre de Vinzelles and patented in this country, has several outstanding features, among them the fact that tubing is produced continuously at the high speed of 6 to 8 ft. per minute, with the possibility of obtaining non-porous tubes able to resist as high an inner pressure as 1500 psi. and even more if required.

The remarkable advantage of the Imbert–de Vinzelles process is that the pipe-making unit is compact enough to be mounted on a truck or trailer and to produce the pipe on location, thus enabling great savings in the costs of transportation and labor. In comparison to steel, a continuous pipe laid by a "pipe-liner" would cost one-third less.

The first Imbert–de Vinzelles machine is presently used as a stationary unit by the Stratitub Company at Elbeuf in Normandy and has proved so successful that the French are now seriously considering the setting up of a similar plant on a ship to lay a projected pipeline across the Mediterranean from Algeria to the French coast.

According to the inventors, layers of Fiberglas fabric, impregnated with polyester resins, are welded together into a rigid tube within a matter of seconds by a new high frequency polymerization method. Usually these tubes contain about an equal amount in weight of fiberglas and of polyester resins. But it is possible to make use of other fibrous materials and to adapt the formula of the thermoplastic resins

to the desired characteristics of the finished product.

In general, the tubes produced by the process fall into two distinct groups: on the one hand the "mechanical" type (porous), mainly for structural purposes (light weight combined with great mechanical strength), chemical applications (resistance to corrosion), and electrical uses (good insulator), and on the other hand, the leakproof "duct" type tubing (nonporous) for carrying almost any sort of liquid or gas under pressure.

The "duct" tubing is perfectly leakproof, because its inner layers are impregnated with a special resin exclusively made for the process, which has the same chemical resistance as polyester and yet has an infinitely greater elasticity. Practically, the tubes have about the same flexibility as steel piping. In weight, however, the Imbert–de Vinzelles tubing is approximately four times lighter than steel tubing of equal mechanical resistance and of the same inner diameter.

Both types of the Imbert–de Vinzelles tubing find a great many applications. Tubing of the "mechanical" type is, for instance, an ideal material for posts, railings, ladders, etc., exposed to a corrosive atmosphere; for light weight but sturdy sheds and movable greenhouses; and for insulating rods of high mechanical strength. Other applications are less obvious. Thus the largest manufacturer of dies for the French automobile industry is using "mechanical" tubing for building up the die models of large body parts: a "forest" of tubes is welded to a base at such angles and with such lengths as to form a strong support for a plastic sheet of the exact shape of the die. These die models are not only cheaper than those made heretofore and much easier to handle, because of their light weight, but they remain strictly constant in their dimensions, as the tubes are unaffected by temperature or by humidity.

The tubing of the "duct" type is usable for most liquids and gases, even those containing hydrogen sulfide for instance, and can be buried in the soil, since neither rodents nor molds attack it. Besides, the inventors have given much attention to the aging of reinforced polyesters, taking into account the studies made in this country by the Air Force Laboratories as well as in Italy by Professor Saccenti of the Montecatini Company. As a result, they have been able to formulate their thermoplastic materials and to take special care in the manufacturing process in order to prevent any undesirable effect of aging. As a matter of fact, no deterioration at all has been found after several years, even on samples submitted to many cycles of accelerated aging. Hence, the Imbert–de Vinzelles "duct" tubing is eminently suitable for nearly any distribution system of liquids and gases, including water, crude oil, natural gas, etc.

Finally, it should be expressly mentioned that the Imbert–de Vinzelles process is not limited to tubing, but applies just as well to the continuous manufacture of other products of indefinite length and constant cross-sections of various shapes.

Throw-Away Hypodermic Needle.—An estimated 10% of the nation's hospitals are now using disposable hypodermic needles and syringes to reduce pain and the danger of infection from injections.

According to Henry Wendt, Jr., general manager of American Cyanamid Company's Surgical Products Division, this trend has developed in the 9 months since Cyanamid and

other manufacturers introduced throw-away hypodermic units.

The needles may be used on existing glass syringes, or in combination with the disposable plastic syringes.

The advantages of the disposable hypodermic unit are that it assures a sharp needle every time and eliminates the danger of improper sterilization.

Dull needles are one of the main causes of pain from injections. Improper sterilization can spread various hepatitis diseases and staphylococcus infection.

To insure against accidental re-sterilization of the new one-time-use products, Cyanamid's disposable needle, sold under the Vim brand name, is fitted to the syringe with a plastic hub.

Both the hub and the plastic syringe will melt if heated, but are resistant to corrosion from medicines.

Cost of the needles alone is less than 5 cents, while the needle and syringe combination sells for less than 20 cents.

Plasma Generator.—The first production model plasma generator to come from Avco's Research and Advanced Development Division, Wilmington, Mass., was recently delivered to NASA's Jet Propulsion Laboratory in Pasadena, Calif. This new laboratory tool is capable of supplying a continuous flow of air at the extreme temperatures and with the pressures characteristic of the hypersonic re-entry of ballistic missiles. The arc is the result of a two-year program in which the Avco Research Laboratory and the Research and Advanced Development Division developed and refined a special electric air arc to test new materials for use on the Air Force Titan and Minuteman nose cones. The arc uses from 20 kilowatts to more than one megawatt of power. Temperatures obtained with the plasma generator have reached 18,000 F.

The value of this type of simulation for new material development was recently demonstrated on a nose cone in a full ICBM flight. Here Avco-developed material was used in a 5000-mile flight of the experimental Thor-Able vehicle at ICBM speeds of 15,000 miles per hour and through very high re-entry temperatures.

The environmental conditions of air enthalpy, pressure and heat flux associated with re-entry can be closely simulated in the laboratory with the Avco plasma generator. The device also has been used for arc-heated wind tunnel experiments.

The generator heats gases to high temperatures by passing them through an electric arc. The gas to be heated is injected tangentially into the arc chamber, flows through the area occupied by the arc discharge, momentarily becomes part of the discharge, and then flows through the throat of the anode into a plenum chamber. In the plenum chamber different portions of the gas mix to produce a greater uniformity of plasma temperature and velocity at the exit nozzle. The working plasma is then discharged into free or controlled atmospheres. Although the unit is normally operated with air it may also operate as required with other gases including helium, argon, and nitrogen.

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